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Cosmological spacetimes from negative tension brane backgrounds

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Abstract: We identify a time-dependent class of metrics with potential applications to cosmology, which emerge from negative-tension branes. The cosmology is based on a general class of solutions to Einstein-dilaton-Maxwell theory, presented in hep-th/0106120. We argue that solutions with hyperbolic or planar symmetry describe the gravitational interactions of a pair of negative-tension q-branes. These spacetimes are static near each brane, but become time-dependent and expanding at late epoch — in some cases asymptotically approaching flat space. We interpret this expansion as being the spacetime's response to the branes' presence. The time-dependent regions provide explicit examples of cosmological spacetimes with past horizons and no past naked singularities. The past horizons can be interpreted as S-branes. We prove that the singularities in the static regions are repulsive to time-like geodesics, extract a cosmological "bounce" interpretation, compute the explicit charge and tension of the branes, analyse the classical stability of the solution (in particular of the horizons) and study particle production, deriving a general expression for Hawking's temperature as well as the associated entropy.

Keywords: Superstrings and Heterotic Strings, p-branes, Cosmology of Theories beyond the SM.

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1. Introduction

The shortest path between two truths in the real domain passes through the complex domain. — J. Hadamard

1.1 Motivation

Recently, there has been considerable interest in the dynamics of brane interactions. The interest was motivated partly by the insights which static brane configurations have already given to long-standing low-energy issues like the hierarchy problem, and partly by the potential application of brane collision/annihilation processes to the cosmology of the very early Universe [1, 2]. These developments have proceeded in parallel with renewed effort toward understanding string theory on time-dependent, cosmological backgrounds [3]–[10].

A remarkable feature, which has emerged from studies of brane physics, is the existence of physically sensible objects with negative tension — a prime example being orientifolds [11]. These objects are expected to bear important implications for cosmology. For example, some negative-tension objects do not satisfy the standard positive-energy conditions which underlie the singularity theorems. As such, they may open up qualitatively new kinds of behaviour for the very early Universe. Negative-tension objects also admit the possibility of zero-tension objects, whose existence may shed light on the origin of the currently small size of the cosmological constant [12]. For instance, an effectively tensionless 3-brane can be constructed by wrapping a combination of Dirichlet and orientifold 5-branes on two small-sized internal dimensions.

It then behooves us to construct a cosmology built out of objects having negative- or zero-tension. In so doing, it is imperative to understand first the large-scale gravitational fields produced by these objects. In this paper, we take a step towards improving our understanding of these objects by providing a class of simple space-times which describe gravitating negative-tension objects, based on the solutions of [4], which describe cosmological spacetimes with a horizon, but with singularities only in the static region of the full spacetime. We will see that these singularities correspond to negative-tension branes of opposite charge which is consistent with the interpretation given in [8] of similar geometries in terms of orientifold planes (although as we will see the singularities in our metrics are not necessarily orientifold planes). We believe these space-times are useful for developing intuition concerning such objects, as they are no more complicated to analyze than the well-known Schwarzschild black-hole. The space-times we discuss in this paper enjoy the following properties:

- They are classical solutions to the combined field equations involving dilaton, metric, $(q+1)$-form ($q \geq 0$) antisymmetric tensor fields. For particular choices of coupling parameters, they are classical solutions to bosonic field equations of supergravity and string effective field theory at low-energy.

- They describe field configurations of a pair of q-branes carrying mutually opposite q-form charge and equal but *negative* tension. These q-branes constitute *time-like* singularities of the space-time metric which are separated from one another by an infinite proper distance.

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We would like to interpret these space-times, containing a pair of negative-tension objects, as counterparts of (higher-dimensional versions of) the so-called C-metric space-time [13] (see also their dilatonic generalizations [14]), describing the behavior of a pair of charged particles in acceleration due to nodal singularities [15]. Unlike the generic C-metric solutions, however, the ones we present here have no nodal singularity, which we interpret as meaning that no additional stress-energy (like the "rods and ropes" of the C-metric) is required in order to induce the negative-tension branes to move along their given world-surfaces.

The space-time is time-independent in the immediate vicinity of each brane. The static nature of the space-time metric may be understood as a consequence of Birkhoff's and Israel's theorems for negative- and zero-tension objects. By contrast, part of the space-time which lies to the future of both branes is time-dependent. The boundary between the two regions — time-independent versus time-dependent regions — is a horizon of the space-time. Curiously, the time-dependent part of the space-time resembles that of recently discussed S-brane configurations [5].

1.2 Negative tension versus stability

As many of the unusual features of these space-times are traceable to the fact that the source carries negative tension, it is worth recalling why such branes are believed to make sense [11] — and potentially to be virtues [16] — in the low-energy world. Traditionally, negative-tension (and negative-mass) objects were considered pathological on the following grounds. Consider the world-volume action of a single q-brane, which has the form

$$S_b = -T \int d^{q+1}y \sqrt{-\det \gamma} + \cdots , \tag{1.1}$$

where T denotes the brane's tension,[1] $\gamma_{ab} = g_{MN}\partial_a x^M \, \partial_b x^N$ refers to the metric induced on brane's world-volume by the space-time metric $g_{MN}(x)$, and the ellipses represent contributions of other low-energy modes of the brane dynamics. If the embedding of the brane were free to fluctuate about some fixed value, x_0^M, in the ambient space-time, then $x^M = x_0^M + \xi^M$ and ξ^M is a *dynamical* variable. A negative-definite value of the tension, $T < 0$, poses a problem since it implies a negative-definite kinetic energy — and hence an instability — for the fluctuation ξ^M [17]. This being so, one always assumes that the tension T of a dynamical object is positive-definite.

The explicit construction of sensible negative-tension objects such as the orientifolds within string theory hints how the aforementioned instability and no-go argument are avoidable. Specifically, the argument does not apply in the instances of the space-time studied in this paper, simply because these objects are not free to move in the ambient space-time. Rather, negative-tension branes are arranged to be localized at special points, such as orbifold fixed points or space-time boundaries, and hence do not carry dynamical variables such as ξ^M, causing an instability as the tension T becomes negative-valued.

[1] Here, we tacitly assume that the object moves relativistically so that the energy density ρ (as measured per unit q-dimensional volume) equals to the tension T.

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The immobility is consistent with the equations of motion because it is the equation of motion for the missing dynamical variables ξ^M which would have required the brane's centre-of-mass to follow a geodesic trajectory (if the brane were neutral).

We believe the immobility of the negative-tension branes helps explaining several otherwise puzzling features of the spacetime we describe in this paper. For instance, as will be shown later, the source branes do not follow geodesics in the spacetime, even when the branes are arranged not to carry any electric charge. On the other hand, despite not following the geodesics, the spacetime contains no nodal singularity. The situation is unlike what arises with the C-metric solution, where the nodal singularities are interpreted as consequences of the external stress-energy which is required to force the sources to move along their non-geodesic trajectories. This kind of stress-energy is not required for negative-tension objects since, by construction, they are not required to move along geodesics in any case. The immobility of these objects might also help explaining why the late-time regions of the metric are time-dependent.[2]

1.3 Outline

This paper is structured in the following way. First, in section 2, we review our solutions, with a particularly simple Schwarzschild-like example, for which a generalization of the Birkhoff and the Israel theorems [18] to negative-tension objects applies. We continue in section 3 to a much more general class of solutions. We also show in this section how special cases of these solutions reduce to various configurations which have been considered elsewhere in the literature. Section 4 supports the interpretation in terms of negative-tension sources in two ways. First, the conserved charges which are carried by the source branes are computed using the curved-space generalizations of Noether's theorem. Second, the response of a test particle to the gravitational field is examined through the study of time-like and null geodesics. Section 5 describes how the throat between the two cosmological regions can be interpreted as a time-like bounce. Section 6 investigates small fluctuations about the solutions, with evidence presented for the instability of some of their remote-past features. We believe the late-time metric to be stable, and we regard the calculations of this section as a first step towards a more comprehensive stability analysis. We also discuss in this section the relevance of the time-like singularities, and why these can make sense of space-time's overall causal structure. In this section, we show that a Hawking temperature can be defined, and we present preliminary arguments that this reflects the spectrum of particles seen by static observers. Finally, we summarize our conclusions in section 7, where we also comment on some future directions for research which our calculations suggest, above all on the construction of the cosmological models.

2. Simple solutions

Before presenting our solutions in their most general form, we pause here first to build

[2] We are largely concerned with *classical* aspects of the negative-tension objects. The stability issue creeps out again once quantum effects such as pair-creation/annihilation of these objects are taken into account. We discuss this issue further in later sections.

intuition by describing their simplest variant: vacuum solution to Einstein's field equation, $R_{\mu\nu} = 0$, in four dimensions.

2.1 Schwarzschild revisited

We start with the well-known Schwarzschild black-hole, whose space-time geometry is given — in the asymptotically flat region $r \geq 2M$ — by:

$$ds_I^2 = -\left[1 - \frac{2M}{r}\right] dt^2 + \left[1 - \frac{2M}{r}\right]^{-1} dr^2 + r^2 \left(\sin^2\theta d\phi^2 + d\theta^2\right), \tag{2.1}$$

whose constant r and t surface is the two-sphere S_2. *Birkhoff's theorem* states that eq. (2.1) is the unique solution for representing *spherically symmetric* non-rotating black holes.[3] *Israel's theorem* [18] states further that eq. (2.1) is also the unique solution for representing *static* non-rotating black holes.[4]

As is well known, the apparent singularity of the metric eq. (2.1) on the surface $r = 2M$ is a coordinate artifact. For $r < 2M$, the metric goes over to that of the interior region, for which the role of r and t gets interchanged, leading to a time-dependent metric of the form:[5]

$$ds_{\rm II}^2 = -\left[\frac{2M}{t} - 1\right]^{-1} dt^2 + \left[\frac{2M}{t} - 1\right] dr^2 + t^2 \left(\sin^2\theta d\phi^2 + d\theta^2\right). \tag{2.2}$$

Note that the surface of constant r and t remains the same two-sphere S_2. A real, *space-like* singularity occurs for $t \to 0$, which is to the future of any observer falling into the Schwarzschild black-hole.

A particularly simple form of the solutions which are of interest in this paper may be obtained from eq. (2.2) by an *analytic continuation*, $\theta \to i\theta$ followed by an overall sign change of the metric.[6] This leads to the following time-dependent vacuum solution:

$$ds_I^2 = -\left[1 - \frac{2P}{t}\right]^{-1} dt^2 + \left[1 - \frac{2P}{t}\right] dr^2 + t^2 \left(\sinh^2\theta d\phi^2 + d\theta^2\right). \tag{2.3}$$

Note that, after the analytic continuation, the surface of constant r and t has turned from the two-sphere, S_2, to the hyperbolic surface, $\mathcal{H}_2$, viz. sign of the curvature scalar is flipped from positive to negative. The metric is explicitly time-dependent but homogeneous otherwise — it has a space-like Killing vector $\xi = \partial_r$ in addition to the symmetries of the hyperbolic surface $\mathcal{H}_2$ at fixed values of r and t.

Equation (2.3) is well-defined for $t > 2P$, but as is clear from its connection with the Schwarzschild black-hole, the degeneracy at $t = 2P$ is merely a coordinate artifact. An extension of the metric to $t < 2P$ is given by performing the same continuation as the one

[3]We emphasize that this theorem assumes nothing regarding time-(in)dependence of the solution.

[4]Although we describe in detail in this section the four-dimensional case, our discussion trivially generalizes to $2 + n$ dimensions — with $n \geq 2$ — through the replacement $1/r \to 1/r^{n-1}$.

[5]We adopt here the convention of always labelling the time coordinate as t, both inside and outside the horizon.

[6]Equivalently we can take $\theta \to i\theta, \phi \to i\phi, t \to ir, r \to it, M \to iP$.

leading to eq. (2.3):

$$ds_{\rm II}^2 = -\left[\frac{2P}{r} - 1\right] dt^2 + \left[\frac{2P}{r} - 1\right]^{-1} dr^2 + r^2 \left(\sinh^2\theta d\phi^2 + d\theta^2\right). \tag{2.4}$$

The metric in this region is static and retains the hyperbolic space $\mathcal{H}_2$ at constant r and t. A real, *time-like* singularity occurs for $r \to 0$, and this is the structure we are primarily interested in this paper.

Just as $r = 2M$ does for the Schwarzschild black hole, the surface $t = 2P$ defines a *non-compact* horizon of the space-time described by eqs. (2.3) and (2.4). This is most transparently seen from the Penrose diagram of the space-time, given in figure 1. It is simply a $\pi/2$-rotation of the Penrose diagram for the Schwarzschild space-time.

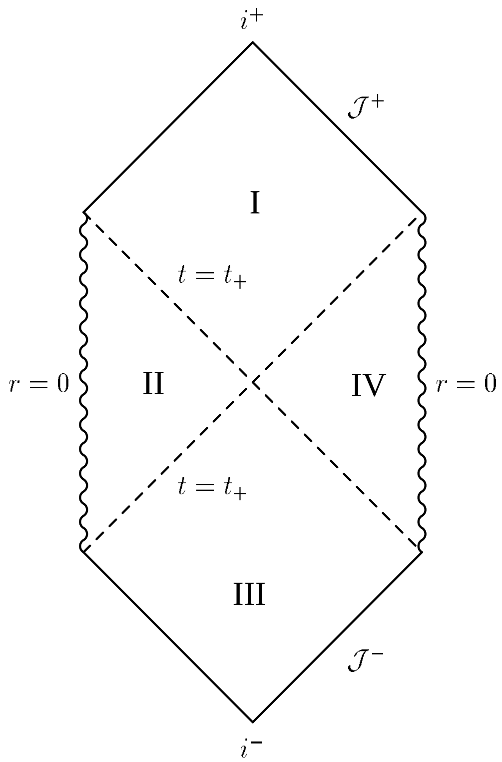

Figure 1: Penrose diagram for the $k = 0, -1$ case. This diagram is very similar to the Schwarzschild black hole (rotated by $\pi/2$), but now region I (III) is not static, but time-dependent with a Cauchy horizon and region II (IV) is static.

An observer in region I experiences a time-dependent, expanding region of the space-time, which becomes *flat* as $t \to \infty$. The observer sees no singularity in null or time-like future, but will experience two time-like singularities in the past. By contrast, an observer in regions II and IV experiences a static space-time, and sees only a single time-like singularity in the past. Observers in region III see no singularities to their past, but have both time-like singularities in their future light cones. On the other hand, observers at fixed values of r, θ and ϕ in the static regions — including the singularities themselves as a limiting case — do not follow geodesics and so experience a proper acceleration.

The above description suggests a viable interpretation for this solution, as well as for many of the other more general ones which we present in subsequent sections. Regions II and IV describe the space-time external to two objects which we argue to be negative-tension branes. These branes may also carry other conserved charges. Region I gives the time-varying transient gravitational fields which these branes produce at late times. Region III similarly describes the time-reversal of this last time-dependent process.

In this interpretation, the horizons, which are reminiscent of S-branes (in a precise sense explained below) [5], describe the locus of instants when observers make the transition from having only one of the branes in their past light cone to having them both in their past.

2.2 Kruskal coordinates

For understanding the overall structure of the above space-time, it is useful to have explicit coordinates for the complete maximally-extended space-time, whose Penrose diagram is pictured in figure 1. Starting from the geodesically incomplete solutions eqs. (2.3) and (2.4), a convenient choice of coordinates for the extended geometry is obtained by using the analog of the Kruskal coordinates, defined as follows:

- For $t \geq r_+$, define

$$\begin{aligned} v &:= \pm \left[\frac{t}{2P} - 1\right]^{1/2} e^{t/4P} \cosh\left(\frac{r}{4P}\right) , \\ u &:= \left[\frac{t}{2P} - 1\right]^{1/2} e^{t/4P} \sinh\left(\frac{r}{4P}\right) , \end{aligned} \tag{2.5}$$

  where the square-root is taken to be positive, and the upper and the lower signs correspond to region I and III, respectively.

- For $r \leq r_+$, define

$$\begin{aligned} v &:= \left[1 - \frac{r}{2P}\right]^{1/2} e^{r/4P} \sinh\left(\frac{t}{4P}\right) , \\ u &:= \mp \left[1 - \frac{r}{2P}\right]^{1/2} e^{r/4P} \cosh\left(\frac{t}{4P}\right) , \end{aligned} \tag{2.6}$$

  where again the positive square-root is understood, and the upper and the lower signs correspond to regions II and IV, respectively.

For the time-dependent regions, I and III, the metric in the Kruskal coordinates becomes

$$ds^2 = \frac{16P^3}{t} e^{-t/2P} (-dv^2 + du^2) + t^2 \left(\sinh^2 \theta d\phi^2 + d\theta^2\right) . \tag{2.7}$$

The Penrose diagram of figure 1 follows from this metric after performing a straightforward conformal transformation which brings the asymptopia in to a finite-distance. The horizon corresponds in these coordinates to the lines $u = \pm v$. Similarly, in the static regions II and IV, where $r < r_+$, we have the relation

$$u^2 - v^2 = \left[1 - \frac{r}{2P}\right] e^{r/2P} . \tag{2.8}$$

The singularity, $r = 0$, is then given by the hyperbola $u^2 = v^2 + 1$. Using these coordinates it is straightforward to see that the proper distance between each singularity and the horizon is infinite, and therefore the two singularities have to be thought to be infinitely separated.

2.3 Alternative analytic continuations

In addition to the one we have adopted, can one find yet another analytic continuations of the Schwarzschild black-hole which would also admit cosmological interpretations in

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the asymptotic regions? Some time ago, Gott [19] (see also [20]) proposed an alternative analytic continuation: For regions I and III, his continuation is the same as ours: $\theta \to i\theta$ followed by an overall signature change. For regions II and IV, his differs from ours and is given simply by $\theta \to \pi/2 + i\tau$ and $t \to i\psi$ applied to eq. (2.1). Thus,

$$\begin{aligned} ds^2_{\mathrm{I,III}} &= -\left[1 - \frac{2M}{t}\right]^{-1} dt^2 + \left[1 - \frac{2M}{t}\right] dr^2 + t^2 \left(\sinh^2\theta d\phi^2 + d\theta^2\right), \\ ds^2_{\mathrm{II,IV}} &= +\left[1 - \frac{2M}{r}\right]^{-1} dr^2 + \left[1 - \frac{2M}{r}\right] d\psi^2 + r^2 \left(\cosh^2\tau d\phi^2 - d\tau^2\right). \end{aligned} \tag{2.9}$$

His analytic continuation was built upon totally different physical motivations from ours, and it was interpreted as giving rise to the space-time associated with a gravitating tachyon. The interpretation was borne out on the ground that the $r = 0$ singularity is time-like and is perpendicular to the space-like world-line of a tachyon, much the same way as the space-like singularity of the Schwarszchild metric is related to the time-like world-line of a massive particle at rest.[7] The existence of a tachyon would signal instability of the space-time involved, and the solution eq. (2.9) in fact describes a "*bubble of nothing*" [10, 21, 22]. Note the difference between this solution and the solution of eq. (2.3). In eq. (2.3), the metric is a warped product of a lorentzian surface, $\Sigma_{1,1}$, parameterized by r, t, and a euclidean hyperbolic space $\mathcal{H}_2$, parametrized by θ, ϕ. By contrast, for the gravitating tachyon, eq. (2.9), the r, ψ coordinates parameterize a euclidean "cigar" (assuming that ψ is periodic), while the τ and ϕ coordinates describe a two-dimensional de Sitter space.[8]

Space-time geometries whose Penrose diagrams are similar to ours have also been studied previously for Einstein gravity coupled to a variety of other fields [15, 23, 24]. In the context of string theory, an example was found in [25] on which the string dynamics is describable in terms of non compact, two-dimensional Wess-Zumino-Novikov-Witten (WZNW) models. More recently, a general class of brane solutions was found [4] in which some of the coordinates parameterize a subspace with constant curvature, labelled by $k = 1, 0, -1$ as for the Friedman-Robertson-Walkers (FRW) metrics. In these solutions, the case $k = 1$ represents the standard black-brane solutions of a system consisting of gravity/dilaton/antisymmetric-tensor fields, but the cases $k = 0, -1$ exhibit the Penrose diagram similar to the one discussed above, viz. Figure 1. We shall see that eqs. (2.3) and (2.4) furnish particular cases of the general solutions of [4]. More recently, geometries similar to ours have been considered as orbifold cosmological models [8] and as S-branes [5, 6, 7] (see also [3]). In particular, interesting cosmological consequences were drawn in [8] from space-time geometries which these authors claim are produced by a pair of orientifolds.

3. General solutions

We now turn to the description of a wider class of solutions which extend the simple considerations of section 2 to various space-time and brane's world-volume dimensions, and to

[7] This interpretation was later reconsidered by Gibbons and Rasheed [22].

[8] We thank Gary Gibbons for a discussion on these points.

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a system involving metric, dilaton, and $(q+1)$-form tensor fields — a system encompassing bosonic fields of diverse supergravity or superstring theories and their compactifications. This wider class of solutions was already obtained in [4], in which the primary interest was generalization of the well-known black branes of string theory to all possible signs of the curvature parameter, k, of the maximally-symmetric transverse space.

3.1 Dilaton-generalized Maxwell-Einstein solutions

The system we will consider is defined by the following Einstein-frame action in $d = (n + q + 2)$-dimensional space-time:

$$S = \int_{\mathcal{M}_d} d^d x \sqrt{g} \left[\alpha R - \lambda (\nabla\phi)^2 - \eta e^{-\sigma\phi} F_{q+2}^2 \right] , \tag{3.1}$$

where $g_{\mu\nu}, \phi, F$ denotes metric, dilaton field, and $(q + 2)$-form tensor field strength, respectively. Stability requires the constants α, λ and η to be positive, and, if so, they are removable by absorbing them into redefinition of the fields.[9] It is nevertheless useful to keep them arbitrary since this would allow to examine various reduced systems, where each constant is taken zero (to decouple the relevant fields) or negative (e.g. to reproduce E-brane solutions in the hypothetical type-II* string theories, related to the type-II string theories via time-like T-duality. See later.). eq. (3.1) includes supergravity, and so also low-energy string theory, for specific choices of d, σ and q (for instance $d = 10$, $q = 1$ and $\sigma = 1$).

The field equations obtained from eq. (3.1) are given by:

$$\alpha G_{\mu\nu} = \lambda T_{\mu\nu}[\phi] + \eta e^{-\sigma\phi} T_{\mu\nu}[F] , \tag{3.2}$$

$$2 \lambda \nabla^2 \phi = -\sigma \eta \, e^{-\sigma\phi} F^2 , \tag{3.3}$$

$$\nabla_\mu \left(e^{-\sigma\phi} F^{\mu\cdots} \right) = 0 , \tag{3.4}$$

where

$$T_{\mu\nu}[\phi] = \nabla_\mu\phi \nabla_\nu\phi - \frac{1}{2} g_{\mu\nu} (\nabla\phi)^2 , \qquad \text{and} \qquad T_{\mu\nu}[F] = (q + 2) F_\mu{}^{\cdots} F_{\nu\cdots} - \frac{1}{2} g_{\mu\nu} F_{\cdots}^2 .$$

We are interested in classical solutions whose space-time geometry take the form of an asymmetrically warped product between q-dimensional flat space-time and n-dimensional maximally-symmetric space, parametrized by a constant curvature $k = 0, \pm 1$. For this ansätz, the solutions depend only on one warping variable — either t or r — and ought to exhibit isometry $\mathrm{SO}(1,1) \times O_k(n) \times \mathrm{ISO}(q)$, where $O_k(n)$ refers to $\mathrm{SO}(n-1,1)$, $\mathrm{ISO}(n)$ or $\mathrm{SO}(n)$ for $k = -1$, 0 and 1, respectively. The ansätz is motivated for describing a flat q-brane propagating in $d = (n+q+2)$-dimensional ambient space-time, where n-dimensional transverse hyper-surface is a space of maximal symmetry, and constitute an extension of Birkhoff's and Israel's theorems.

[9] The canonical choices are $\lambda = \alpha = 1/2$, and $\eta = 1/2(q + 2)!$ in units where $8\pi G = 1$.

A solution satisfying these requirement is readily obtained as [4]

$$ds^2 = h_-^A \left(-h_+ h_-^{1-(n-1)b} dt^2 + h_+^{-1} h_-^{-1+b} dr^2 + r^2 h_-^b dx_{n,k}^2\right) + h_-^B dy_q^2 \,, \tag{3.5}$$

$$\phi = \frac{(n-1)\sigma b}{\Sigma^2} \ln h_- \,, \tag{3.6}$$

$$F_{try_1\ldots y_q} = Q\epsilon_{try_1\ldots y_q} r^{-n}, \qquad \epsilon_{try_1\ldots y_q} = \pm 1 \,. \tag{3.7}$$

The notations are as follows. The metric of an n-dimensional maximally symmetric space, whose Ricci scalar equals to $n(n-1)k$ for $k = 0, \pm 1$, is denoted as $dx_{n,k}^2$. The harmonic functions $h_\pm(r)$ depend on two first-integral constants, $r_\pm$, and are given by:

$$h_+(r) = s(r)\left(1 - \left(\frac{r_+}{r}\right)^{n-1}\right), \qquad h_-(r) = \left|k - \left(\frac{r_-}{r}\right)^{n-1}\right|, \tag{3.8}$$

where

$$s(r) = \operatorname{sgn}\left(k - \left(\frac{r_-}{r}\right)^{n-1}\right). \tag{3.9}$$

The constant Q is given by

$$Q = \left(\frac{4n(n-1)^2\alpha\lambda(r_+ r_-)^{n-1}}{(q+2)!\,\eta\,(\alpha n\Sigma^2 + 4(n-1)\,\lambda)}\right)^{1/2}, \tag{3.10}$$

where Σ and b are constants defined in terms of parameters of the action as

$$\Sigma^2 = \sigma^2 + \frac{4\lambda}{\alpha}\frac{q(n-1)^2}{n(n+q)}, \qquad b = \frac{2\alpha n\Sigma^2}{(n-1)(\alpha n\Sigma^2 + 4(n-1)\lambda)}.$$

Likewise, the exponents A, B in eq. (3.5) are defined in terms of the same parameters as

$$A = -\frac{4\lambda q(n-1)^2 b}{\alpha n(n+q)\Sigma^2}, \qquad \text{and} \qquad B = \frac{4\lambda(n-1)^2 b}{\alpha(n+q)\Sigma^2} = -\frac{n}{q}A\,.$$

The solution defined by eqs. (3.5)–(3.7) is unique modulo trivial field redefinitions: $\phi \to \phi(r) + 2\phi_0$ and $F \to Fe^{\sigma\phi_0}$, which in turn can be compensated by rescaling of the space-time coordinates and the first-integral constants, $r_\pm$.

The two first-integral constants, $r_\pm$, are related intimately to two conserved charges associated with the solution. One of these is the q-form electric charge Q — see eq. (3.7) — acting as the source of the $(q+2)$-form tensor field strength. The electric charge is measurable from the flux integral $\oint {}^*F_{q+2}$ over the n-dimensional symmetric space. The explicit integral yields the electric charge given precisely by eq. (3.10), so is a function of the first-integral constants. For the q-brane to be physically sensible, the electric charge Q ought to be real-valued. From eq. (3.10) and from the stability condition $\eta > 0$,[10] the condition renders an inequality $(r_- r_+)^{n-1} \geq 0$. Note that $r_- = 0$ if and only if $Q = 0$, and the point $r = r_-$ is potentially singular (or a horizon) only if $k = 1$. The second conserved charge is associated with the Killing vector of the metric, and so can be understood as a mass, in a sense which will be made explicit later.

[10] This last conclusion does not follow for E-branes, for which η may be chosen negative (see later sections).

A dual, magnetically charged solution is obtainable from the electrically charged solution by making the duality transformation: $F_{q+2} \to \tilde{F}_n = {}^*F_{q+2}$, $\sigma \to -\sigma$ and $q \to (d-4-q)$ in eqs. (3.5), (3.6) and (3.7), where F_{q+2} is related to $\tilde{F}_n$ through the dilaton-dependent expression:

$$F_{q+2} = \mathrm{e}^{\sigma\phi}\epsilon_{q+2,n}\tilde{F}_n\,. \tag{3.11}$$

The solution presented above is expressed as a function of the coordinate r. One readily finds that r denotes a spatial coordinate for $k = -1, 0$ in so far as $r < r_+$. For $r > r_+$, the harmonic function h_+ flips the overall sign, so the r coordinate becomes temporal. As such, we will relabel the coordinates as $r \leftrightarrow t$ for $r > r_+$ so that t labels always the time coordinate.

Drawing lessons from the simple solution presented in section 2, we are primarily interested in $k = -1, 0$ cases. Note that the $k = -1$ solution is obtainable from the $k = 1$ solution in much the same way via the following *analytic continuation*:

$$\begin{aligned} t &\longrightarrow ir\,, \qquad r \longrightarrow it\,, \qquad \Omega_n \longrightarrow i\Omega_n\,, \qquad \text{and} \\ r_+ &\longrightarrow ir_+\,, \qquad r_-^{n-1} \longrightarrow -(ir_-)^{n-1}\,. \end{aligned}$$

Note that this is precisely the same as that defined the simpler, Schwarzschild-type solution in the previous section. See footnote 6. The above procedure also suggests that we can obtain yet another solution with $h_+ = \left|k - (r_+/r)^{n-1}\right|$ and $h_- = 1 - (r_-/r)^{n-1}$ via an alternative analytic continuation $r_- \to ir_-$ and $r_+^{n-1} \to -(ir_+)^{n-1}$.[11] It turns out these new solutions are singular at $r = r_-$ for generic values of the parameters. As such, they would correspond to more standard cosmology evolving from a past singularity. Further new (and generically singular) solutions are also obtainable by T-dualizing the above solutions with respect to the coordinate r in the time-dependent region and t in the static region. In this case the corresponding element of the metric — g_{rr} or g_{tt} — in the string frame gets inverted and the dilaton field is shifted accordingly (see for instance, ref. [27]).

3.2 Asymptotic and near-horizon geometries

For foregoing discussions, we pause here to examine both the asymptotic and the near-horizon geometries of our solution, eqs. (3.5)–(3.7). As the $k = 1$ case parallels to the standard black-brane studies, we focus primarily on the $k = 0, -1$ cases. For the ease of the analysis, we adopt the *isotropic coordinates*, defined by

$$\tau^{n-1} = \left(t^{n-1} - r_+^{n-1}\right). \tag{3.12}$$

The near-horizon and the asymptotic limits then correspond to $\tau \to 0$ and $\tau \to \infty$, respectively.

The metric eq. (3.5) takes, in the isotropic coordinates, the form:

$$ds^2 = \left(\frac{H_-}{H_+}\right)^{A+b}\left[-\frac{H_+^{2/(n-1)}}{H_-}d\tau^2 + \frac{H_-^{1-nb}}{H_+^{2-nb}}dr^2 + \tau^2 H_+^{2/(n-1)}dx_{n,k}^2\right] + \left(\frac{H_-}{H_+}\right)^B dy_q^2\,. \tag{3.13}$$

[11] The additional minus sign is required to ensure the real-valuedness of the electric charge Q.

The harmonic functions $H_\pm(\tau)$ are given, for the $k=-1$ case, by

$$H_+ = 1 + \left(\frac{r_+}{\tau}\right)^{(n-1)}, \qquad H_- = H_+ + \left(\frac{r_-}{\tau}\right)^{(n-1)}, \tag{3.14}$$

and, for $k=0$ case, by

$$H_+ = 1 + \left(\frac{r_+}{\tau}\right)^{(n-1)}, \qquad H_- = \left(\frac{r_-}{\tau}\right)^{(n-1)}. \tag{3.15}$$

Likewise, the dilaton field and the $(q+2)$-form tensor field strength are given in the isotropic coordinates by

$$\phi(\tau) = \frac{(n-1)\sigma b}{\Sigma^2} \ln\left(H_+^{-1} H_-\right), \qquad \text{and} \qquad F_{try_1\ldots y_q}(\tau) = Q\epsilon_{try_1\ldots y_q} \tau^{-n} H_+^{-n/(n-1)}. \tag{3.16}$$

From eqs. (3.13) and (3.16), we now analyze the limiting geometries for the two cases $k=-1,0$ separately.

The $k=-1$ brane: in the asymptotic region, $\tau\to\infty$, and both H_+ and H_- approach the unity. That is, the asymptotic geometry is flat:

$$ds^2|_{\text{asymptotic}} = -d\tau^2 + dr^2 + \tau^2 d\mathcal{H}_n^2 + dy_q^2, \tag{3.17}$$

where $d\mathcal{H}_n^2 = dx_{n,-1}^2$. Moreover, both the dilaton field and the $(q+2)$-form field strength become zero: $\phi, F\to 0$. This is very interesting as the time-dependent regions I and III tend asymptotically to a vacuum state corresponding to (a patch of) flat space-time, both in the asymptotic past and future infinity.

In case the system under consideration is the bosonic part of a supersymmetric theory, the asymptotic region could constitute a supersymmetric vacuum. For instance, as the asymptotic geometries are flat, in- and out-states might be defined naturally having anywhere up to the maximal number of unbroken supersymmetries. Clearly, cosmologies with asymptotic supersymmetry could have many interesting features.

In the near-horizon region, $\tau\to 0$, and the harmonic functions are reduced to

$$H_+ \longrightarrow \left(\frac{r_+}{\tau}\right)^{n-1}, \qquad \text{and} \qquad H_- \longrightarrow \left(\frac{\overline{r}}{\tau}\right)^{n-1},$$

where $\overline{r}^{n-1} := (r_-^{n-1} + r_+^{n-1})$. For simplicity, consider a particular choice of the parameters so that $r_- = r_+$ — the result does not change if they are different. The metric then behaves as

$$ds^2|_{\text{near-horizon}} = -d\tilde{t}^2 + \left(\frac{\tilde{t}}{r_+}\right)^2 dr^2 + r_+^2 d\mathcal{H}_n^2 + dy_q^2, \tag{3.18}$$

where unimportant numerical factors are absorbed by rescaling coordinate variables, and the time coordinate is newly defined as $\tilde{t} := \tau^{(n-1)/2} r_+^{(n-3)/2} 2^{(1-A-b)/2}$. Note that the near-horizon geometry does not depend explicitly on the dimension n of the transverse space. Moreover, the dilaton field and the $(q+2)$-form field strength tend to constants in this limit.

We thus find that the near-horizon geometry of the time-dependent regions I and III ($t > r_+$) is described by the direct product of a two-dimensional Milne Universe with coordinates $\tilde{t}$ and r, an n-dimensional hyperbolic space with coordinates x_n, and a q-dimensional flat space with coordinates y_q. In the near-horizon geometry, the Penrose diagram of figure 1 goes over to that of the Milne Universe, illustrated in figure 2. The apparent singularity at $\tilde{t} = 0$ is harmless, as it corresponds to a regular point at the horizon.

Alternatively, the near-horizon limit can be taken from the static interior regions — regions II and IV of figure 1. In this case, we find that the two-dimensional spacetime with coordinates r, t is reduced to Rindler spacetime — the shaded region in figure 2.

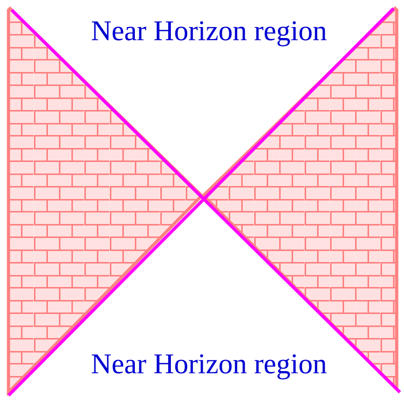

Figure 2: Penrose diagram for the near horizon limit. It is simply the Milne Universe, where the shaded zone is excluded and an apparent singularity sits at $\tilde{t} = 0$.

The $k = 0$ brane: the $k = 0$ branes exhibit several marked differences from the $k = \pm 1$ ones. The main difference is in the asymptotic geometry, which in this case does not become flat as $\tau \to \infty$. In particular, in this limit, the coefficient of $d\mathcal{E}_n^2 = dx_{n,0}^2$ goes to zero and the dilaton field runs logarithmically to $\phi = -\infty$.

The result for the metric in the near-horizon limit -again taking $r_- = r_+$ for simplicity — is:

$$ds^2|_{\mathrm{near-horizon}} = -d\tilde{t}^2 + \left(\frac{\tilde{t}}{r_+}\right)^2 dr^2 + r_+^2 d\mathcal{E}_n^2 + dy_q^2 \,. \tag{3.19}$$

As might be expected starting from the original causal structure, the geometry is again a direct product of a two-dimensional Milne Universe, an n-dimensional flat space and a q-dimensional flat space. The Milne Universe geometry of the near-horizon region seems to be quite generic for all these solutions. Note, however, that, in this case, the near horizon geometry is an exactly flat space-time, in contrast to the $k = -1$ brane.

3.3 Special cases

Recently, variants of the supergravity/superstring brane solutions were considered in the literature. We now pause to show that they are nothing but particular cases of our brane solution eqs. (3.5)–(3.7).

Black-branes: $k = +1$ the $k = 1$ branes were studied extensively in the literature, and was interpreted as a black q-brane [26]. For the special case $q = 0$ and in the absence of the dilaton field coupling, the solution reduces (as it should) to a higher-dimensional version of the Reissner-Nordström black-hole. It is possible to define the Arnowitt-Deser-Misner (ADM) mass for the black q-brane, and turns out to be given by $M = r_+^{n-1} + r_-^{n-1}[1 - (n - \sigma^2/\Sigma^2)b]$.

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It was known that, in the case the dilaton coupling is non zero, that the surface $r = r_+$ is a coordinate singularity corresponding to an event horizon, while the surface $r = r_-$ is a *bona fide* null singularity of the scalar curvature. This singularity is formed when the unstable inner horizon of the Reissner-Nordström black-hole is perturbed and made singular by coupling the black-hole to the dilaton field. Only if $r_- < r_+$, is this singularity covered by the horizon, otherwise it is naked. The point $r = 0$ is also singular in the usual sense.

Note that, in eq. (3.5), the region beyond the null singularity at $r = r_-$ is well defined. As is known, an extremal black-brane is obtained with the choice $r_+ = r_-$, which also promotes the symmetry of the space-time to $\mathrm{SO}(n + 1) \times \mathrm{SO}(q, 1)$. The solution then corresponds to the field due to a Dq-brane of string theory, for which the horizon becomes singular.[12] An exception to the above statement, for which the solution *is* well-defined and the dilaton field well-behaved, is the case $q = 3$.

S-branes: $k = 0, -1$: consider next the $k = 0, -1$ branes. These resemble the analytic continuation of the Schwarzschild black-hole considered in section 2, whose Penrose diagram is given by figure 1. First of all, note that, for fixed r and t, the sign of the harmonic function h_+ flips as one changes from $k = +1$ to $k = -1, 0$. This implies that roles played by r and t coordinates are interchanged — see eqs. (3.5), (3.8) and (3.9) — and the metric is time-dependent. In this case, the point $t = r_-$ is a regular point, while $t = r_+$ is a *past Cauchy horizon*. See figure 1. The point $r = 0$ is now a time-like singularity, which is behind the past horizon of the future time-dependent region I. Unlike the case of the Schwarzschild-type solution, this singularity can be avoided by future-directed time-like curves in the region between the horizon and the singularity. See figure 3.

A simple analysis indicates that, for $k = -1, 0$, there is no real- and positive-valued choice for r_+ and r_-, for which the solution would be "extremal" in the sense of displaying enhanced symmetries. In this case, the maximal symmetry is just the symmetry assumed for the ansatz, as discussed in the previous section.

This last fact is an important difference between our solutions and the S-brane solutions discussed in [5, 6, 7]. The symmetries considered in these works differ from those we assume, as they imposed $\mathrm{SO}(n, 1) \times \mathrm{ISO}(q+1)$ symmetry [5, 6]. Only for the $q = 0$ case, symmetries of our solution yields the same as those discussed in [5, 6, 7]. In the absence of the dilaton field, our solutions coincide with the S-branes for any n (again with $q = 0$).

E-branes: $\eta < 0$: our solutions also have counterparts in the literature (in various limits) if one makes non-standard choices for the signs of α, η and λ. In this case, we obtain the euclidean branes, or E-branes, discussed in [28]. This connection can be seen by taking "wrong" sign kinetic term for the $(q + 2)$-form tensor field: $\eta < 0$. With this choice, Q remains real *provided* we also take $(r_- r_+)^{n-1} < 0$. This leads to the low energy limit of truncated type-II* string theories [28]. Taking $\eta < 0$, one can take an *extremal limit*: $r_-^{n-1} = -r_+^{n-1}$ in the original solution eq. (3.5), with a real-valued electric charge, but in the type-II* string theories. Note that, in this limit, the horizon becomes singular (similar

[12]Of course, this singularity is outside the domain of validity of the low-energy string effective field theory.

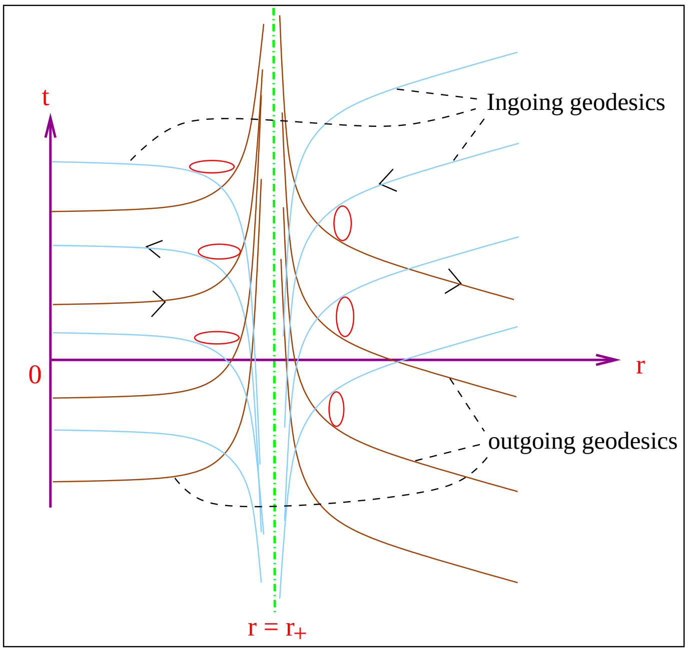

Figure 3: Null-like geodesics form in the simple case, $b = 0$, $n = 3$.

to what happens for extremal black-branes). However, for the $E(3+1)$-brane, the horizon is regular and the dilaton field is well-behaved, in close analogy with the D3-branes in type-IIB string theory.

In this case, the harmonic function H_- is reduced to 1, rendering the space-time geometry as:

$$ds^2 = H_+^{(2/(n-1)-b-A)} \left(-d\tau^2 + \tau^2 d\mathcal{H}_n^2\right) + H_+^{-B} \left(dr^2 + dy_q^2\right). \tag{3.20}$$

This solution interpolates between flat spacetime in the asymptotic region, $\tau \to \infty$, and direct product, $dS_{q+2} \times \mathcal{H}_n$, of a $(q+2)$-dimensional de Sitter space and an $n-$dimensional hyperbolic space in the near-horizon region, $\tau \to 0$. The $E(3+1)$-brane is particularly simple, whose metric takes the form:

$$ds^2|_{E(3+1)} = -\left(\frac{r_+}{\tau}\right)^2 d\tau^2 + \left(\frac{\tau}{r_+}\right)^2 dy_{q+1}^2 + r_+^2 d\mathcal{H}_n^2. \tag{3.21}$$

The above extremal solutions correspond exactly to the E-branes of the type-II* string theories. One then expects that there ought to be non-extremal E-brane solutions as an equivalent of the black branes in the original type-II string theories. Some of these may be constructed for type-II* string theories as above, by taking both r_-^{n-1} and η negative-valued. This yields an analog of the black-branes in type-II* string theories. In the non-extremal case, r_- is again singular. One can further find other solutions of the type-II* string theories for $k = 1$ by taking more general values for the first-integral constants and for the coupling parameters.

4. Interpretation I: negative tension brane

In this section, we shall be drawing a viable interpretation of our solution eqs. (3.5)–(3.7). We will first investigate in further detail the two conserved charges alluded in section 2. We will see that, while the definition of the electric charge of the source object do not pose problems, the definition of the gravitational mass requires careful treatment. We will then explore the spacetime geometries and causal structures by studying geodesic motion of a test particle.

4.1 Conserved quantities

We start by identifying two conserved quantities as Noether charges carried by the source branes, whose metric, dilaton field, and $(q+2)$-form field strength are given as in eqs. (3.5)–(3.7).

Electric charge. We have argued earlier that the constant Q, eq. (3.10), defined roughly by a flux integral of the Poincaré dual n-form field strength ${}^*F_{q+2} = \tilde{F}_n$ over the n-dimensional maximally symmetric space, is interpretable as a conserved electric charge. We now elaborate the argument, and associate the electric charge with q-branes located at each of the two time-like singularities.

From the field equation eq. (3.7) of the $(q+2)$-form tensor field strength, a conserved charge density can be defined through $d^*F_{q+2} = {}^*J$. This leads to the following expression for the electric charge:

$$Q = \int_\Sigma d\Sigma_{\mu i \ldots} \nabla_\nu \left(e^{-\sigma\phi} F^{\mu\nu i \ldots} \right) = \int_{\partial\Sigma} d\Sigma_{\mu\nu i \ldots} e^{-\sigma\phi} F^{\mu\nu i \ldots} \,, \tag{4.1}$$

where Σ refers to any $(n+1)$-dimensional space-like hyper-surface transverse to the q-brane. Advantage of the above expression of the electric charge lies in the observation that the integrand vanishes almost everywhere by virtue of the field eq. (3.4). It does not vanish literally everywhere, however, because the integrand behaves like a delta function at each of the two time-like singularities. Conservation of Q is also clear in this formulation, as the second equality of eq. (4.1) exhibits that Q is independent of Σ so long as the boundary conditions on $\partial\Sigma$ are not changed.

Evaluating the flux integral eq. (4.1) over a space-like hyper-surface t = constant within either of the two static regions (regions II and IV of figure 1), we retrieve the result eq. (3.10), up to an overall normalization, for the electric charge at each of the two time-like singularities. The electric charge turns out *equal* but *opposite* for each of the q-branes located at the two time-like singularities in the fully extended space-time: $Q_{\rm II} = -Q_{\rm IV}$. One can draw this conclusion by directly applying eq. (4.1) to a choice of the space-like hyper-surface Σ, which extends from the immediate right of the singularity located in region II to the immediate left of the singularity located in region IV, and passes through the "throat" where these regions touch (see figure 1). As this choice of the hyper-surface does not enclose the singularities, the flux integral in eq. (4.1) necessarily vanishes. This implies that the (outward-directed) electric fluxes through the two components of the boundary, $\partial\Sigma = \Sigma_{\rm II} + \Sigma_{IV}$, are equal and opposite to one another, and so the same is true for the electric charges which source the dilaton and the tensor fields on the two boundaries.

We are led in this way to identify the conserved quantities, $\pm Q$, with electric charges carried by each of the two q-branes located at the time-like singularities. Which brane carries which sign of the electric charge may be determined as follows. As eq. (3.7) defines the constant Q relative to a coordinate patch labelled by r and t, the key observation is that the coordinate t can increase into the future only for one of the two regions, II or IV. Then, the charge $+Q$ applies to the brane whose static region t increases into the future, and $-Q$ applies to the brane whose t increases into the past.

Gravitational mass. Recall that the metric eq. (3.5) is static only in the regions II and IV, but not in the regions I and III. This means that only in the static regions is it possible to define a conserved gravitational mass (or tension) in the usual sense for the branes located at time-like singularities.

A procedure for evaluating the gravitational mass in the present situation is to adopt the *Komar integral* formalism [35], which cleanly associates a conserved quantity with any Killing vector field, ξ^μ, by defining a flux integral:[13]

$$K[\xi] := c\alpha \oint_{\partial\Sigma} dS_{\mu\nu} D^\mu \xi^\nu \,. \tag{4.2}$$

Here, c denotes a normalization constant, and Σ is again an $(n+2)$-dimensional space-like hyper-surface transverse to the q-brane, and $\partial\Sigma$ refers to the boundary of Σ. The Komar charge K is manifestly conserved since it is invariant under arbitrary deformations of the space-like hyper-surface Σ for a fixed value of the fields on the boundary $\partial\Sigma$.

The connection between the flux integral eq. (4.2) and the more traditional representation of $K[\xi]$ as an integral over Σ of a current density is obtained by using the identity $D^2\xi^\mu = -R^\mu{}_\nu \xi^\nu$ and Gauss' law:

$$K[\xi] = 2c\alpha \int_\Sigma dS_\mu D_\nu D^\mu \xi^\nu = \int_\Sigma dS_\mu J^\mu(\xi) \,; \tag{4.3}$$

where the current density

$$J^\mu(\xi) = c\left(T^\mu_\nu \xi^\nu - \frac{1}{d-2} T^\lambda_\lambda \xi^\mu\right), \tag{4.4}$$

is conserved in the sense that $D_\mu J^\mu = 0$. This last expression utilizes the properties of Killing vector fields, as well as Einstein's equations for relating $R_{\mu\nu}$ to the total stress-tensor, $T_{\mu\nu}$. As we see explicitly later, if $T_{\mu\nu}$ is non zero, then the value taken by K can depend on the location of the boundary $\partial\Sigma$ in eq. (4.2).

We now argue that, if we adopt the Komar integral for the definition of the q-brane tension $\mathcal{T}$, the *sign* of the tension ought to be the *same* for both static regions, II and IV. This is most transparently seen for the Schwarzschild-like solution for which $T_{\mu\nu} = 0$, by applying the definition of eq. (4.2) to the two-component boundary of a surface, Σ_t,

[13] We thank Gary Gibbons for interesting discussions on this section.

of constant t. The boundary extends from near the singularity in region II over to near the singularity in region IV. Then, the vanishing of $T_{\mu\nu}$ leads to the conclusion that the contribution from each boundary component is equal and opposite: $K_{\rm II}(\partial_t) = -K_{\rm IV}(\partial_t)$. However, since the globally-defined time-like Killing vector is only future-directed in one of the two regions, II or IV, local observers will identify $\mathcal{T} = -K[\partial_t]$ in the region where ∂_t is past-directed, leading to the conclusion $\mathcal{T}_{\rm II} = \mathcal{T}_{\rm IV}$.

To evaluate the tension $\mathcal{T} = K[\partial_t]$ in the patch for which ∂_t is future-directed, we will choose for the hyper-surface Σ a constant-t spatial slice and for the boundary $\partial\Sigma$ a $r = r_0$ (viz. a constant radius) slice in the regions II and IV, respectively. It turns out that, if $Q \neq 0$, the expression for the tension depends on the value r at which the boundary $\partial\Sigma$ is defined. This is also true for the radius-dependent mass of the Reissner-Nordström black-hole. Likewise, we would expect that the gravitational mass of the q-brane depends on the stress-energy of the $(q+2)$-form tensor field for which the brane is a source if $Q \neq 0$. Explicitly, we find the tension is given by:

$$\begin{aligned} \frac{\mathcal{T}(r)}{V} &= -2\alpha(n-1)\left[r_-^{n-1} - kr_+^{n-1} + r_-^{n-1}(2+A-(n-1)\,b)\left(\left(\frac{r_+}{r}\right)^{n-1} - 1\right)\right], \\ &= -\frac{(n-1)}{8\pi G}\left[r_-^{n-1} - kr_+^{n-1} + \frac{2Q^2}{(n+q)(n-1)}\left(\frac{1}{r^{n-1}} - \frac{1}{r_+^{n-1}}\right)\right], \end{aligned} \tag{4.5}$$

where the normalization constant c has been chosen to ensure that $\mathcal{T}/V$ takes the conventional (positive) value for the black-brane solutions ($c = 4$), and V is the volume of the $(n+q)$-dimensional hyper-surface over which the integration is performed. The standard normalization choices $16\pi G\eta = 1/(q+2)!$ and $16\pi G\alpha = 1$ are made in the second equality above.

Recall that for $k = -1, 0$ in the static part of the space-time r must satisfy $r \leq r_+$, and this shows that the tension as defined above is negative throughout the static region. For the special case of the simple Schwarzschild-type solution discussed in section 2, the tension becomes simply $\mathcal{T}/V = M/V = -P/V_n$, where V_n denotes the finite volume of the n-sphere (so $V_2 = 4\pi$). As the Schwarzschild case is a vacuum space-time ($T_{\mu\nu} = 0$), this result is independent of the choice of r, and is only non zero due to the δ-function singularity in $T_{\mu\nu}$ which the solution displays as the time-like singularities are approached. This again shows how the tension may be identified with q-branes sitting at these singularities. For $k = 1$ we recover the standard charge dependence of the tension, in this case the calculation is done in the region inside the second horizon.

Negative tension, $\mathcal{T} < 0$, for both branes is in accord with the form of the Penrose diagram of figure 1, which, in the static regions, II and IV, is similar to the Penrose diagram for a negative-mass Schwarzschild black-hole [36], or the overcharged region of the Reissner-Nordström black-hole. As we shall see next, negative-valued gravitational mass or tension is also borne out by the behavior of the geodesics of a test particle in these regions.

Note that the Komar integral technique used above can also be used to compute a "conserved" charge in the time-dependent regions, which may be relevant for confirming the S-brane interpretation of the horizons of these regions. The quantity obtained in this way involves the ti-components of the stress-tensor, and defines a generalized momentum

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corresponding to the symmetry under shifting r in this region. We regard however the existence of the static regions, where conserved quantities like tension can be clearly defined, as being very helpful in providing a physical interpretation of geometries like S-branes.

4.2 Repulsive geodesics

To substantiate why the negative-tension interpretation is a viable one, we will study geodesic motion of a test particle in the background of the solution eqs. (3.5)–(3.7). Specifically, we will be primarily interested in the $k = -1$ case, and study geodesic motion of a massless or a massive test particle, which couples only to the metric but not to the dilaton or the $(q + 1)$-form tensor fields. To understand the nature of the solution beyond the static regions, we will follow the geodesic motion of these particles starting from the past time-dependent region III, passing through the static regions II and IV, and eventually ending in the future time-dependent region I.

Null geodesics. In the static regions II and IV, the radial coordinate ranges over $r < r_+$, so we consider the *radial* null geodesics defined by $ds^2 = d\mathcal{H}_n^2 = dy_q^2 = 0$. This implies that:

$$- h_-^{A+1-(n-1)b} h_+ \dot{t}^2 + h_-^{A-1+b} h_+^{-1} \dot{r}^2 = 0 \,. \tag{4.6}$$

where the dots refer to differentiation with respect to the affine parameter along the worldline. Thus,

$$\frac{dt}{dr} = \frac{\dot{t}}{\dot{r}} = \pm h_+^{-1} h_-^{-1+nb/2} \,, \tag{4.7}$$

where the $\pm$ sign is for outgoing/ingoing geodesics.

As the regions II and IV are time-independent, a first-integral of the geodesic equation renders energy conservation:

$$- \xi_m \dot{x}^m = h_-^{A+1-(n-1)b} h_+ \dot{t} = E \,, \tag{4.8}$$

where $\xi = \partial_t$ denotes the time-like Killing vector. eqs. (4.8) and (4.6) together furnish

$$\dot{r} = \pm E h_-^{-A+(n-1)b/2} \,. \tag{4.9}$$

Though we have derived them in the static region, eqs. (4.9) and (4.7) are applicable equally well in other regions too. One can integrate them numerically for a generic initial condition, and we illustrate the result in figure 3. The outgoing geodesics are those which travel outside the horizon and pass into the region I of figure 1. Similarly, ingoing geodesics are those which come from the past time-dependent region III in figure 1.

Since $h_\pm \to \infty$ as $r \to 0$, there is no difficulty for integrating either eq. (4.9) or (4.7) right down to $r = 0$, indicating that null geodesics reach the singularities in a finite interval of both affine parameter and coordinate time.

Time-like geodesics. Radially directed time-like geodesic motion is characterized by $\dot{s}^2 = -1$, $d\mathcal{H}_n^2 = dy_q^2 = 0$, and so:

$$- h_-^{A+1-(n-1)b} h_+ \dot{t}^2 + h_-^{A-1+b} h_+^{-1} \dot{r}^2 = -1 \,. \tag{4.10}$$

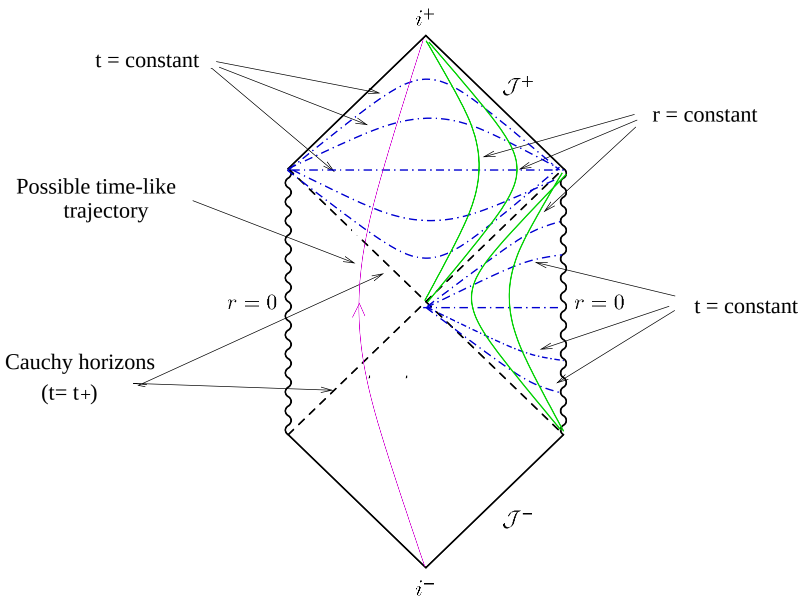

Figure 4: Typical time-like trajectory. Shown also are constant-r, constant-t surfaces and the Cauchy horizon of the solution.

Combining this with the first-integral of energy conservation, eq. (4.8), we find the following condition for time-like geodesics in all regions:

$$\dot{r} = \pm \left(E^2 h_-^{-2A+(n-2)b} - h_-^{-A-b+1} h_+ \right)^{1/2}, \tag{4.11}$$

where again the sign is + for outgoing and − for incoming radial time-like geodesics.

As before, the geodesic equations can be integrated numerically in the general case, but it is clear that the observer takes a finite proper-time to reach the horizon ($h_+ \to 0$), across which the observer can pass freely. In terms of the coordinate time, we have:

$$\frac{dt}{dr} = \pm \frac{E h_-^{(n-1)b-A}}{h_+ h_- \sqrt{E^2 h_-^{-2A+(n-2)b} - h_-^{1-A-b} h_+}}. \tag{4.12}$$

The integral of eq. (4.12) diverges as $h_+ \to 0$, so we see that it takes an infinite time for a particle to reach the horizon as seen by a static observer inside the horizon ($r < r_+$).

On the other hand, as $r \to 0$, $h_\pm \sim 1/r^{n-1} \to \infty$. In this limit, the first term inside the square root of eq. (4.11) grows slower than the second term, which renders the square-root to become complex-valued if r becomes sufficiently small. We see from this that an infalling time-like geodesic never hits the singularity. Instead the infalling observer reaches a point of closest approach, $r_m > 0$, at which the square root in eq. (4.11) becomes zero, and reflected outward. The turning point for a time-like geodesic is given by the value r_c of the coordinate r, for which the following expression holds:

$$r_c = \frac{r_+}{(1 + E^2 [h_-(r_c)]^{(n-1)b+A-1})^{1/(n-1)}}. \tag{4.13}$$

Note that r_c is always between $0 \leq r_c \leq r_+$. For example, for $r_- = 0$, and $k = -1$, we have that

$$r_c = \frac{r_+}{(1+E^2)^{1/(n-1)}}\,. \tag{4.14}$$

Gravitational repulsion. We see from the above considerations concerning geodesic motion of a test particle that the two time-like singularities act gravitationally as *repulsive* centers, as no infalling time-like geodesic can hit them.[14] In this sense, the space is time-like (although not null) geodesically complete. Observers who originate in the remote past — region III — enter one of the static regions by passing through the past horizon, and then leave this through the future horizon of the future time-dependent region, I. This resembles what happens in other geometries, such as the Reissner-Nordström black-hole.

Putting together the above results, we are led to draw the following conclusion. Static observers in regions II and IV are those whose world trajectories follow lines of constant r, and these observers have proper accelerations which are directed towards the nearest singularity. We find the following expression for the proper acceleration, for $k = -1, 0$ and $q = 0$. We have, in the coordinates adopted,

$$a^r = -\frac{(n-1)h_+h_-^{1-b}}{2r^n}\left[\frac{r_+^{n-1}}{h_+} + \frac{r_-^{n-1}}{h_-}\right], \tag{4.15}$$

so the value of the acceleration is always negative.

The singularities themselves are special instances of these observers for whom $r \to 0$, in which limit the proper acceleration becomes infinitely large. As discussed in the Introduction, this behavior does not contradict with the equations of motion for the branes at the singularities since for negative-tension branes these do *not* imply motion along a geodesic (or otherwise) within the space-time.

This is in contrast to what is found for accelerating positive-mass particles, as described by the C-metric. For this metric, the particle world-lines are also not geodesics, so the particles follow trajectories which are not self-consistently determined by the fields which the particles source. For positive-mass particles this inconsistency shows up through the appearance of nodal defects, which are conical singularities along the line connecting the two particles. These singularities are interpreted as being the gravitational influence of whatever additional stress-energy is responsible for the particle motion [14, 15].

5. Interpretation II: time-like wormhole

Comparison with the Schwarzschild black-hole permits another interpretation of our solution. After re-expressing our solutions eqs. (3.5)–(3.7) in conformal frame, the geometry of the n-dimensional slices turns out that of a *time-like bounce*. On the other hand, the scale factor for the r coordinate resembles an object localized in time, and so is a kind of *time-like kink*. Such bounce/kink behavior would help explain what precisely the S-brane configuration is.

[14] However, infalling null geodesics can hit the singularity.

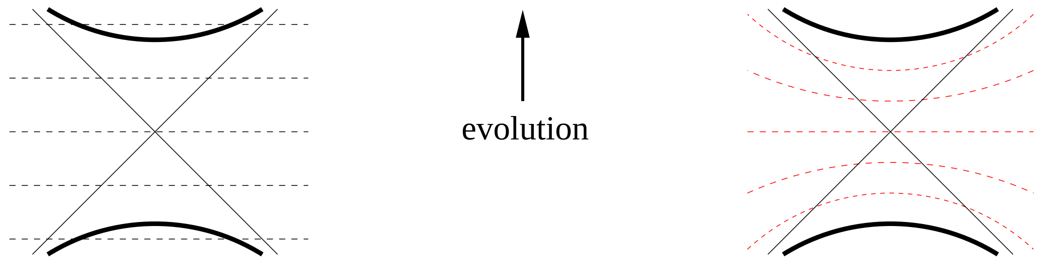

Figure 5: Two possible foliation of the maximally extended space-time of the Schwarzschild black-hole. Both cases lead to the Einstein-Rosen bridge connecting the two static regions.

We shall be interested in foliating the geometry with respect to the time in the time-dependent regions, I and III. We will be finding that the geometry exhibits bounce and kink behavior for the symmetric space and the radial direction, respectively.

5.1 Einstein-Rosen wormhole: a review

We begin by recapitulating the interior dynamics of the Schwarzschild black-hole relevant for our foregoing discussions.

Consider the maximally extended space-time of the Schwarzschild black-hole. We are interested in describing time-evolution of the space-time geometry. We may foliate the space-time as a stack of constant t surfaces. Then, the space-time at sufficiently early epoch consists of two disconnected asymptotically flat components, each containing a space-like singularity surrounded by a past horizon. The two components evolve and, at some early epoch, the two singularities join together and smooth out by forming a "wormhole" connecting the two components. The wormhole neck widens, reaching a maximal proper size $r = 2M$ at the time-symmetric point $t = 0$. This is the instance when the wormhole neck is instantaneously static and the event horizon of the two components join instantaneously. Evolving further, the wormhole neck recontracts, eventually pinching off as the two singularities reappear and the space-time disconnects.

Two remarks are in order. First, as it is evident from the Kruskal coordinates, the process of wormhole formation and recollapse occurs so rapidly that it is impossible to traverse the wormhole and communicate between the two asymptotic regions without encountering the singularity. Second, the picture of the time-evolution depends on the foliation. Consider, for instance, an alternative foliation illustrated in the right of figure 5. In this case, the geometry starts as a space-like singularity in the asymptotic past, grows out as a hyperboloid, reaches a *maximal* neck size of the hyperboloid, and recollapses to a space-like singularity in the asymptotic future. See figure 6 for the comparison.

A natural question is whether a foliation similar to the Schwarzschild black-hole is possible for our solution as well. We find that it is, although a marked difference would be that the time-evolution is with respect to the regions outside the horizon (inside out compared to the Schwarzschild black-hole case) and details of the evolution are somewhat different for $k = -1$ and $k = 0$ branes, although the properties we end up finding turn out similar. As such, we will again separate the discussion for the $k = 0$ and $k = -1$ cases.

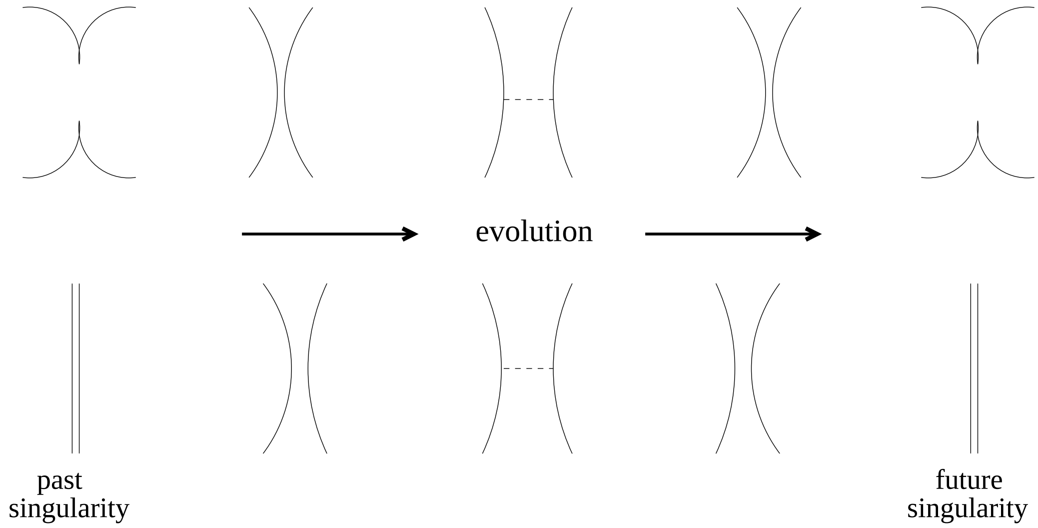

Figure 6: Cartoon view of time evolution of the Einstein-Rosen bridges. The upper/lower sequence corresponds to the evolution for the left/right choice of the foliation in figure 5.

5.2 The $k = -1$ brane

Consider, for simplicity, the case $r_- = 0$ and $q = 0$, for which the singularities are point-like.[15] Recall that the metric in the original coordinates is

$$ds^2 = -\frac{1}{h_+}dt^2 + h_+\,dr^2 + t^2 dx^2_{n,-1}\,, \tag{5.1}$$

where $h_+ = 1 - (r_+/t)^{n-1}$. We now rewrite this metric in terms of the conformal time η (not to be confused with the normalization constant used in earlier sections) as

$$ds^2 = C^2(\eta)\left[-d\eta^2 + dx^2_{n,-1}\right] + D^2(\eta)dr^2\,, \tag{5.2}$$

where the conformal time is defined by

$$C(\eta) = t(\eta) = r_+ \cosh^{2/(n-1)}\left[\frac{(n-1)}{2}\eta\right] \geq r_+\,, \tag{5.3}$$

and so η ranges over $-\infty < \eta < \infty$. Then, the scale factor for the r-direction becomes

$$D(\eta) = \tanh\left[\frac{(n-1)}{2}\eta\right] \tag{5.4}$$

and has the same functional dependence for all values of n. These expressions exhibit the bouncing structure of the $(n+1)$-dimensional space and the (time-like) kink structure of the radial dimension. We illustrate the behavior of the scale factor in figure 7 for the example of $n = 3$ and $r_+ = 1$.

[15]The case $r_- \neq 0$ for $b = 0$ can also be integrated analytically, but gives rise to a more complicated result.

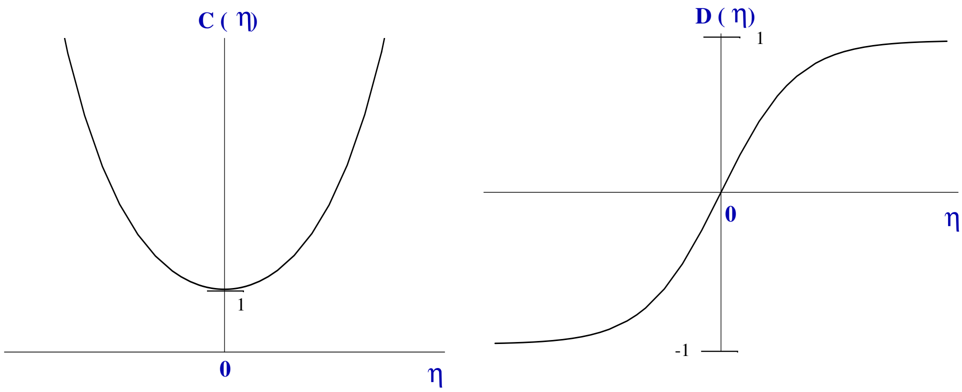

Figure 7: A bounce and kink for the $k = -1$ brane.

5.3 The $k = 0$ brane

In this case, we cannot take a vanishing charge ($r_- = 0$), as then h_- would vanish too. We instead concentrate on the limit $b = 0$ and $q = 0$ but $r_- \neq 0$. The starting metric has the same form as eq. (5.1) and h_+ has the same form also, but now $h_- = (r_-/t)^{n-1}$. In terms of the conformal time η, the metric becomes, as before,

$$ds^2 = C^2(\eta) \left[-d\eta^2 + dx^2_{n,0}\right] + D^2(\eta)dr^2 \,, \tag{5.5}$$

but now with the conformal time defined by

$$C(\eta) = t(\eta) = r_+ \left[1 + \frac{(n-1)^2 r_-^{n-1}}{4r_+^{n-1}}\eta^2\right]^{\frac{1}{n-1}} \geq r_+ \,, \tag{5.6}$$

and ranging again over $-\infty < \eta < \infty$. The scale factor for r is similarly obtained for general n, and is

$$D(\eta) = \frac{2(n-1)r_-^{(n-1)}\eta}{4r_+^{n-1} + (n-1)^2 r_-^{(n-1)}\,\eta^2} \,. \tag{5.7}$$

We see once again the bounce behavior of the $(n+1)$-dimensional symmetric space and the kink behavior of the scale factor for the r-direction. We can see this clearly in figure 8, where as before we plot an example for $n = 3$. Note that, at $\eta = 0$ viz. at $t = r_+$, nothing special happens to the $(n+1)$-dimensional subspace, but the scale-factor for the "extra dimension", r, degenerates to zero!

5.4 Cosmological bounce/kink and time-like wormhole

As anticipated, the cosmological bounce behavior of our solution offers yet another physical interpretation: our solution is reminiscent of a *time-like* version of the Schwarzschild wormhole or Einstein-Rosen bridge, which connects the two asymptotically flat regions in the maximally extended Kruskal coordinate space-time. Our solution corresponds to a $\pi/2$-rotation of the foliation illustrated in figure 5 in the sense that the two time-dependent

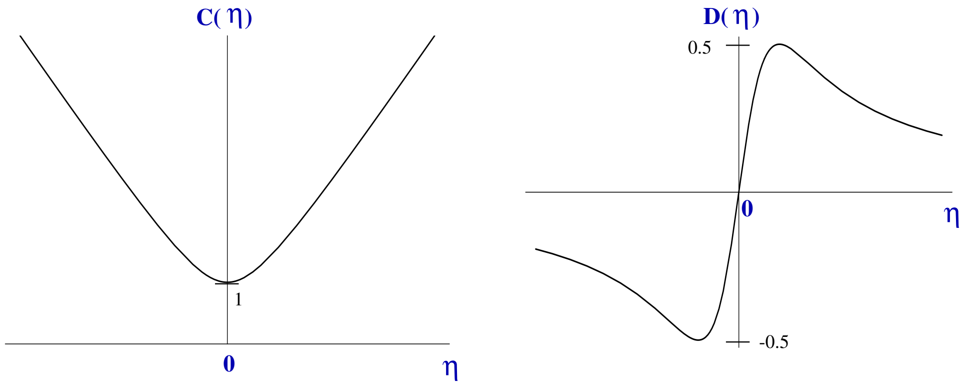

Figure 8: A bounce and kink for the $k = 0$ brane.

regions — instead of the static regions — are connected by a *time-like wormhole*. Note that, according to figures 7 and 8, the geometry of each fixed r slice start out contracting, reaching the minimum volume, and subsequently expanding.

The bounce/kink interpretation of our solution fits also nicely with the interpretation that, in particular cases, our solution reduces to the S-brane (as alluded earlier), and with the proposal that the S-branes are time-like kinks. Our solution clarifies the proposal further in that the S-brane is in fact located at the horizon r_+.

5.5 Comparison with Reissner-Nordström black-hole

An attentive reader would have not missed the resemblance between our solutions and (part of) the spacetime of the Reissner-Nordstrom black-hole. More specifically, if we let the outer horizon of the Reissner-Nordstrom black-hole go to infinity, then the geometry and the Penrose diagram of the two space-times are the same.

We believe that the previous interpretation of the Kruskal diagram for our $k = 0, -1$ solutions in terms of interactions due to negative-tension objects remains valid also for the non-extremal Reissner-Norsdröm black-hole in four dimensions, whose horizon is given by S_2 and $k = 1$ (switching off the dilaton, see [29, pg. 158]). In our analysis, the time-dependent region, the region between the inner and the outer horizons, is interpretable as a destabilization of the space-time due to the combined gravitational field of two negative-mass objects. Inspecting the Penrose diagram of the non-extremal Reissner-Norsdröm black-hole, one notes that the same considerations are applicable. First of all, the two singularities in the region inside the inner horizon, where the space-time is static, still exhibit *opposite charges* and *equal* but *negative masses*. The negative value of the mass obtained from the Komar integral calculation is essentially due to contributions coming from the electromagnetic field.

The past light-cone of an observer in the "time-dependent" region — the region between the inner and the outer horizons — is aware of both the negative-mass objects located inside the inner horizons: the simultaneous repulsion of the two objects propel the observer toward increasing values of the coordinate r. Once the observer crosses the horizon corresponding to $r = r_+$, entering into the region outside the outer horizon, the observer's past light-cone does not see any longer two negative-mass objects, but only one. The interaction with only one object is not sufficient to destabilize the space-time. In the asymptotically flat static region outside the outer horizon, the Komar integral calculation gives a positive mass object: indeed, the effect of the electromagnetic field is suppressed in comparison with the gravitational one.

Passing to a conformal frame, constructing the wormhole solution connecting the time-dependent regions of the metric, one finds a "bounce structure" with a periodic cosine dependence, instead of the hyperbolic-cosine one obtained for $k = -1, 0$, describing in this way a cyclic universe (for related ideas see for instance [30]).

6. Stability, singularity and thermodynamics

An immediate question is whether our solution eqs. (3.5)–(3.7) is stable. In this section, for definiteness, we shall be taking again the particular solution: $k = -1$ brane with $q = 0$, and make a first step toward the complete stability analysis, both at classical and quantum levels. At the same time, based on these results, we draw definitive statements concerning the physical nature of the time-like singularities inherent to our solution.

6.1 The Cauchy horizon

An analysis of the stability of — or the particle production by — a given space-time starts with initially-small fluctuations of the fields involved, and propagates them forward in time throughout the space-time. The set-up therefore presupposes that the initial-value problem is well-posed. In the space-time of eq. (3.5), this is not clear as there exists a *Cauchy horizon*, which separates the past time-dependent region III from the static regions II and IV. The Cauchy horizon exists because initial conditions specified in region III do not uniquely determine the future evolution of the fluctuation fields. They do not do so because all points after the Cauchy horizon have at least one singularity in their past light cone, and so can potentially receive signals from these singularities. This implies that a unique time evolution of a field fluctuation from the past time-dependent region III into the future time-dependent region I must also involve a specification of some sort of boundary condition at the location of the two time-like singularities.

From the perspective of brane physics, the existence of such Cauchy horizons is physically reasonable. Imagine that the time-like singularities are the positions of real branes. There then exists a possibility that these branes might emit radiation into the future time-dependent region I, and the possible choices for boundary conditions at the singularities simply encode the possible emission processes which can occur on branes' world-volume. A well-posed time-evolution problem in the embedding space-time thus requires specification as to whether or not the branes are emitting or absorbing radiation.

When necessary, we shall choose the simplest possible brane boundary condition: we assume the brane neither emits nor absorbs any radiation.

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6.2 The Klein-Gordon equation

We first consider the Klein-Gordon equation for a scalar field propagating in the background eqs. (3.5)–(3.7), with particular attention paid to these equations' limiting behavior at asymptotic infinity, and near the horizons. We then explore some relevant properties of the solutions in these regions.

Consider the Klein-Gordon equation of a massive scalar field:

$$-\frac{1}{\sqrt{g}}\partial_M\left[\sqrt{g}g^{MN}\partial_N\right]\psi+M^2\psi=0$$

in the time-dependent regions I and III. Adopting the isotropic coordinates, the equation is given by

$$-\frac{1}{\sqrt{g}}\partial_\tau\left[\sqrt{g}g^{\tau\tau}\partial_\tau\right]\psi-g^{rr}\partial_r^2\psi-\frac{1}{\omega^2\sqrt{h}}\partial_i\left[\sqrt{h}h^{ij}\partial_j\right]\psi+M^2\psi=0\,. \tag{6.1}$$

Here, for clarity, we denote $h_{ij}(x)$ for the metric on the n-dimensional maximally-symmetric hyperbolic space $\mathcal{H}_n$, whose coordinates are x^i, and write $g_{ij}(\tau,x)=\omega^2(\tau)h_{ij}(x)$. The relevant metric components are:

$$\begin{aligned}
g_{\tau\tau}&=-\left(\frac{H_-}{H_+}\right)^b\frac{H_+^{2/(n-1)}}{H_-}\,,\\
g_{rr}&=\left(\frac{H_-}{H_+}\right)^{1-(n-1)b}\frac{1}{H_+}\,,\\
\omega^2&=\tau^2\left(\frac{H_-}{H_+}\right)^b H_+^{2/(n-1)}\,.
\end{aligned} \tag{6.2}$$

The functional form of the metric involved permits separation of variables, so we take $\psi(r,t,x)=\mathrm{e}^{iP\,r}f(t)\,L_K(x)$, where P and K are separation constants determined by the eigenvalue equations:

$$-\partial_r^2e^{iP\,r}=P^2e^{iP\,r}\,,\qquad\text{and}\qquad-\frac{1}{\sqrt{h}}\partial_i\left[\sqrt{h}h^{ij}\partial_j\right]L_K=K^2L_K\,.$$

Both eigenvalue equations can be solved explicitly, and delta-function or $\mathcal{L}_2$ normalizability of the solutions require both $P^2\geq 0$ and $K^2\geq 0$. The temporal eigenvalue equation then becomes:

$$-\frac{1}{\sqrt{g}}\frac{d}{d\tau}\left[\sqrt{g}g^{\tau\tau}\frac{df}{d\tau}\right]+\left[g^{rr}P^2+\frac{K^2}{\omega^2}+M^2\right]f=0\,. \tag{6.3}$$

Asymptotic past/future. In the asymptotic future and past regions I and III, $\tau\to\infty$, so the metric becomes flat with $H_\pm\to 1$, and the mode functions go over to standard forms. In this limit, eq. (6.3) is reduced to

$$\ddot{f}+\frac{n}{\tau}\dot{f}+\left(P^2+M^2+\frac{K^2}{\tau^2}\right)f=0\,, \tag{6.4}$$

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where the dots represent derivatives with respect to τ. The solution is expressible in terms of the Bessel functions:

$$f(\tau) = \tau^{(1-n)/2} \left[\alpha_1 J_y(\rho\,\tau) + \alpha_2 Y_y(\rho\,\tau)\right], \tag{6.5}$$

where $y = -\frac{1}{2}\sqrt{(n-1)^2 - 4\,K^2}$, the α_1, α_2 are constants of integration, and the parameter in the argument is $\rho = \sqrt{P^2 + M^2}$.

At future infinity in the time-dependent region I (or past infinity in region III), we find the asymptotic behavior of the solution is $f(\tau) \sim \tau^{-n/2} e^{\pm iP\tau}$, if $P \neq 0$. If $P = 0$ then $f(\tau) \sim \tau^{a_\infty}$, with

$$a_\infty = -\frac{1}{2}\left[(n-1) \pm \sqrt{(n-1)^2 - 4K^2}\right]. \tag{6.6}$$

These solutions are oscillatory for all $K^2 > \frac{1}{4}(n-1)^2$, and do not grow with τ for large τ so long as $K^2 \geq 0$.

Near-horizon limit.[16] Near the horizon, $\tau \to 0$ and the asymptotic form is governed by the limits $H_+ \to (r_+/\tau)^{n-1}$ and $H_- \to (\overline{r}/\tau)^{n-1}$, with $\overline{r}^{n-1} = (r_-^{n-1} - k r_+^{n-1})$. The metric functions therefore reduce to $g_{\tau\tau} \to \alpha_\tau \tau^{n-3}$, $g_{rr} \to \alpha_r \tau^{n-1}$ and $\omega \to \alpha_\omega$. The precise values of the constants α_τ, α_r and α_ω are not required, apart from the following ratio:

$$\frac{\alpha_\tau}{\alpha_r} = r_+^2 \left(\frac{\overline{r}}{r_+}\right)^{(nb-2)(n-1)} = r_+^2 \left[\left(\frac{r_-}{r_+}\right)^{n-1} - k\right]^{nb-2}. \tag{6.7}$$

With these limits, the Klein-Gordon equation becomes, in the near-horizon limit:

$$\ddot{f} + \frac{1}{\tau}\dot{f} + \left[\frac{\alpha_\tau P^2}{\alpha_r}\frac{1}{\tau^2} + \alpha_\tau \tau^{n-3}\left(M^2 + \frac{K^2}{\alpha_\omega^2}\right)\right] f = 0\,, \tag{6.8}$$

If $P \neq 0$, then the solutions are oscillatory, having the form $f \sim \tau^{a_0}$, with $a_0 = \pm iP\sqrt{\alpha_\tau/\alpha_r}$. If $P = 0$, then a similar argument shows that the solutions are non singular as $\tau \to 0$.

The logarithmic singularity which is implied by the form τ^{a_0} found above has a familiar source, which is most easily seen by transforming to "tortoise" coordinates: $t_* = t + r_+ \log[(t/r_+) - 1]$, whose range is $-\infty < t_* < \infty$, with $t_* \to -\infty$ corresponding to the horizon due to the logarithmic singularity as $t \to r_+$. In terms of the tortoise coordinate, the dominant part of the Klein-Gordon equation governing the r and t_* dependence of ψ becomes

$$\left(-\partial_{t_*}^2 + \partial_r^2\right)\psi = 0\,.$$

This simple wave equation describes waves propagating in *both* directions across the horizon. Note that the mass term drops out of these asymptotic expressions, and so, near the horizon, a massive field behaves like a massless one, approximately propagating along the light-cone. Just as for our discussion of the geodesics, these ingoing and outgoing modes describe motion into and out of the static regions, II and IV, evolving from the past time-dependent region III and to the future time-dependent region I.

[16] In this subsection, we relax the restriction to $k = -1$, and treat all possible cases on equal footing.

6.3 Classical stability

We may now ask whether our solutions eqs. (3.5)–(3.7) are classically stable in the time-dependent regions, I and III. Classical instability is understood here to mean that initially-small fluctuations grow much more strongly with time than does the background metric. Although a complete stability analysis is beyond the scope of this paper, we perform the first steps here for scalar fluctuations which are governed by the Klein-Gordon equation. For simplicity, we focus in this discussion on the massless case, $M = 0$.

There are two parts to be studied for the stability analysis. First, identify the modes which grow uncontrollably, and then determine whether well-behaved initial conditions can generate the uncontrollably growing modes, if these exist. In the present instance, we have just seen that the asymptotic forms for the Klein-Gordon solutions do not include any growing modes, due to the conditions $P^2 \geq 0$ and $K^2 \geq 0$, which follow from the normalizability of the spatial mode functions.

Potentially more dangerous are growing metric modes near the past horizons, which divide the past time-dependent region III from the static regions II and IV. These are more dangerous because of the infinite blue-shift which infalling modes from the region III would experience as they fall into the horizon. This blue-shift boosts their energy (as seen by infalling observers) to arbitrarily large values, and one suspects that such large energy densities drive runaway behavior in the gravitational modes, much as has been found to be so for the *inner horizon* of the Reissner-Nordström black-hole. Naively we might have expected that the horizon in our case could be better behaved than the Reissner-Nordström case [29, 31] due to the presence in that case of the asymptotically flat static region from where the signals sent to the horizon are infinitely blue-shifted. However in our case that region is absent. Nevertheless this does not guarantee the stability of the horizon and a careful stability analysis needs to be performed.

As a preliminary estimate of whether such an instability does exist, we compute the energy, $E = -u^m \partial_m \psi$ of the Klein-Gordon modes considered above as seen by an observer whose velocity, $u = M\partial_t + N\partial_r$, is well-behaved as it crosses the horizon. The normalization condition $u^2 = -1$ in the vicinity of the horizon allows a determination of how M and N must behave as $\tau \to 0$ (in isotropic coordinates) in order to remain non singular. We find in this way $u^2 \sim -\alpha_\tau M^2 \tau^{n-3} + \alpha_r N^2 \tau^{n-1}$, which is regular near $\tau \to 0$ provided $M \sim \tau^{-(n-3)/2}$ and $N \sim \tau^{-(n-1)/2}$ near the horizon. With this choice, one then finds

$$-E = M\partial_\tau \psi + N\partial_r \psi \sim \psi \tau^{-(n-1)/2} \,. \tag{6.9}$$

Using the asymptotic solution found below eq. (6.8): $\psi \sim \tau^{a_0}$ with $a_0 = \pm i P\sqrt{\alpha_\tau/\alpha_r}$, we see that $E \to \infty$ as the horizon is approached. This suggests that the stress-energy density of the mode under consideration diverges as well in this limit. As such, this mode is likely to destabilize the metric modes near the past horizon, much like what is found for the Reissner-Nordström black-hole near $r = r_-$. Notice that if the horizon were stable, we would have a counter-example to the strong version of the cosmic censorship hypothesis, since observers coming from the past cosmological region III could examine the singularity without having to fall into it (see for instance [32]).

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There is a second kind of instability of the Reissner-Nordström black-hole, which our solutions do *not* share. This second stability problem for the Reissner-Nordström horizon is seen as soon as the Einstein-Maxwell system is extended to include also a scalar field, e.g. Einstein-dilaton-Maxwell system: in this case, the inner horizon turns into a genuine singularity. A similar problem does not arise for our solution, since our solution is already a solution to the combined Einstein-dilaton-$(q+2)$-form Maxwell system. We can see explicitly that turning the dilaton on or off does not change the structure of the horizon. Of course, a more detailed calculation of the metric modes is required to establish definitively whether this instability does really arise.

We see in this way that the horizons to the past of the static regions are likely to be unstable to becoming singularities in response to small perturbations. On the other hand, we do not expect a similar instability for the horizons to the future of the static regions. Certainly, a more detailed stability analysis of these space-times is desirable.

6.4 Issue of quantum stability

Before proceeding describing some aspects of particle production on these spacetimes, we first pause to remind the reader of some general stability issues.

The Hawking-Ellis vacuum stability theorem. Hawking and Ellis [29] have proposed a generalization to curved space of the familiar flat-space stability condition that a system's energy must be bounded from below. They propose that the energy of a physically sensible theory should be required to satisfy the following positivity conditions, at least on classical macroscopically averaged scales. Specifically, for an arbitrary future-directed, time-like unit-vector, t^μ, the corresponding energy flux vector $E^\mu = -T^{\mu\nu}t_\nu$ ought to be null- or time-like and future-directed:

$$|E^\mu|^2 \leq 0\,, \qquad \text{and} \qquad -E^\mu t_\mu = T^{\mu\nu}t_\mu t_\nu \geq 0\,. \tag{6.10}$$

This last inequality implies that the energy density seen by all observers is non negative.

Physically, this condition ensures the vacuum is stable against the spontaneous pair creation of positive- and negative-mass objects. Given that our solution is interpreted here in terms of objects — more precisely, a pair of equal-tension q-branes — whose tensions clearly violate the weak energy condition, one might be concerned about instability due to runaway particle production.

It is important in this kind of discussion to distinguish carefully between the energy density defined by the stress tensor of the fields of the problem, and the tension of the q-branes which are their sources. For field fluctuations it is the local field stress energy which is important, and although the q-brane tension is negative, the field stress energy is everywhere positive or zero. For instance, the simple, four-dimensional Schwarzschild-type solution studied in section 2 has a vanishing energy-momentum tensor except at the location $r = 0$. The geodesically complete spacetime of the solution, however, does not include this point, implying that the energy condition is satisfied *globally*.

Further insight is provided by the consideration of the non-extremal Reissner-Nordström black-hole in four dimensions, the situation elucidated in section 4.3. There, we have shown that the region inside the outer horizon exhibits precisely the same physical characteristics as our solutions: the region between the outer and the inner horizon is cosmological, while the region inside the inner horizon corresponds to the static region, and the black-hole singularity inside the static region is *time-like*. We have argued that the Komar mass is negative if measured inside the static region, i.e. inside the inner horizon. The negative-mass, however, does not imply violation of the energy condition. This is because, as is well-known, the stress-tensor of the electromagnetic field is well-behaved everywhere, and can be related to the the local mass $M(r)$ via, for example, $dM(r)/dr = 4\pi r^2 T_{tt}$. Thus, though T_{tt} is positive everywhere, the local mass $M(r)$ can become negative inside the inner horizon because the large electromagnetic field digs up a deep gravitational potential well. The latter is precisely what renders the Komar mass negative when measured inside the inner horizon. By the same line of reasoning, one can understand why the Komar mass turns out positive *if* measured outside the outer horizon.

Indeed, we have a situation similar to the above cases: the stress-tensor of matter fields in the right-hand side of eq. (3.2) are well-defined, and are positive-definite. Despite being so, the Komar mass, defining a local mass, can become negative inside the horizon, as the positivity of the matter stress-tensor imposes the positivity of radial variation of the tension but not that of the tension itself.

Absence of pair production. The above arguments based on the Schwarzschild-type solution and the non-extremal Reissner-Nordström black-hole interior indicate that the energy condition and hence the vacuum stability condition are satisfied by the more general solution, eqs. (3.5)–(3.7). Nevertheless, it would be highly desirable to find yet another argument for the vacuum stability. We believe that the existence of the orientifolds in string theory point to a possible resolution to the question. The orientifolds, being a class of objects carrying negative tension, are vulnerable to the vacuum stability theorem as well, yet they are perfectly well-behaved structures in string theory. A good example is the orientifold 6-plane in type-IIA string theory. In the limit the eleventh dimension opens up, the orientifold goes over to the Kaluza-Klein monopole involving the transverse Atiyah-Hitchin metric. In the same spirit, we expect that our solutions describe well-defined objects once embedded in higher-dimensional gravity theories or string theories.

An issue is whether it is possible to create a pair of positive- and negative-tension q-branes. The aforementioned vacuum stability theorem is to ensure that such a process cannot possibly take place. In the case of orientifolds, pair creation of orientifold and anti-orientifold would be impossible simply because the boundary condition at asymptotic infinity does not match with that of the vacuum — flat Minkowski space-time. In the case of our solution, the asymptotic geometry at past or future infinity reduces either to Milne Universe for $k = -1$ or to degenerate space-time. Either way, they do not result in a flat Minkowski space-time *globally*. This implies that, even if we may manufacture pair creation of positive and negative-tension q-branes, the process does not lead to a vacuum instability simply because the pair-creation geometry cannot be glued smoothly to the flat Minkowski space-time. It is in this sense, we believe, that the negative-tension objects inherent to our solution leave the vacuum stability theorem unaffected even at non-perturbative level.

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6.5 How singular is the time-like singularity?

We now examine the behavior of waves near the time-like singularity at $r = 0$, and ask whether the singularity is ameliorated when it is probed by waves rather than by particles.[17]

This sort of the problem has been studied previously [33, 34] in the context of static space-times having time-like singularities. In some cases, it can happen that space-times which appear singular when probed by classical particles are not singular when these test particles are treated quantum mechanically as waves. Qualitatively, this occurs when an effective repulsive barrier is produced that does not permit the particles to enter into the singularity, and instead scatters them. More precisely, the singular region is not singular to waves if these waves propagate through the singularity in a definite and unique way. As explained in [34], mathematically, this condition is equivalent to the condition that the time-translation operator for the waves must be *self-adjoint*. A sufficient condition to ensure this property is if only one of the two linearly-independent solutions to the equation

$$D^\mu D_\mu \psi \pm i\psi = 0\,, \tag{6.11}$$

is square-integrable.

In the present case, let us examine the solutions to the massless Klein-Gordon equation near the singularity $r = 0$, where the equation becomes equivalent to eq. (6.11). The condition of non integrability of a solution translates into the following condition on the wave function's radial part, $f(r)$:

$$\parallel f \parallel^2 \propto \int_0 dr r^n h_+ h_- \left(\frac{df}{dr}\right)^2 \longrightarrow \infty\,, \tag{6.12}$$

as $r \to 0$.

Since the Klein-Gordon equation reduces, for r near 0, to:

$$f'' - \frac{(n-2)}{r} f' = 0\,, \tag{6.13}$$

the two independent solutions to this equation behave as

$$f(r) \sim c_0 + c_1 r^{n-1}\,, \tag{6.14}$$

for any dimension n, with arbitrary constants c_0, c_1. It is clear that both of these solutions are normalizable, implying the singularity is wave-singular.

6.6 Temperature and entropy

Given the explicit time dependence of the space-time in the time-dependent regions I and III, one would expect particle production takes place in these regions. This radiation would indicate a quantum instability for the future region. A calculation of this radiation is beyond the scope of the present work, but we will make a preliminary analysis which shows that a Hawking temperature can be associated with the static regions II and IV of the space-time.

[17] We limit our discussion to the massless case: for the massive one, the singularity is already well behaved.

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Hawking temperature. An indication that some observers may see excitations with a thermal spectrum is offered by adopting the Hartle-Hawking computation of the Hawking temperature for a black-hole [36]. These steps also lead to the definition of a Hawking temperature for the space-time under consideration, when applied to the static regions II and IV.

The estimate proceeds by performing a euclidean continuation of the metric in this region by sending $t \to i\tau$, and then demanding no conical singularity at the horizon in this euclidean space-time. This condition requires the euclidean time coordinate to be periodic $\tau \sim \tau + 2\pi/\kappa$, and so implicitly defines a temperature: $T = \kappa/(2\pi)$.

The r- and τ-dependent parts of the euclidean metric in the static region are:

$$\begin{aligned} ds_E^2 &= |h_+|^{-1} h_-^{A+b-1} dr^2 + |h_+| h_-^{A+1-(n-1)b} d\tau^2 \,, \\ &\approx h_-^{A+b-1} \left(\frac{r_+}{(n-1)\rho} \right) d\rho^2 + h_-^{A+1-(n-1)b} \left(\frac{(n-1)\rho}{r_+} \right) d\tau^2 \,, \\ &\equiv dR^2 + \kappa^2 \, R^2 d\tau^2 \,, \end{aligned} \tag{6.15}$$

where $\rho = r_+ - r \ll r_+$ gives the coordinate distance from the horizon. The last equality of eqs. (6.15) defines the rescaled radial coordinate R and the parameter

$$\kappa = \frac{(n-1)}{2r_+} h_-^{1-nb/2} \,, \tag{6.16}$$

which determines the temperature. Here, $h_- = h_-(r_+) = |k - (r_-/r_+)^{n-1}|$ denotes the value of this quantity at the horizon.

We find in this way the temperature:

$$T = \frac{\kappa}{2\pi} = \frac{n-1}{4\pi r_+} \left| k - \left(\frac{r_-}{r_+} \right)^{n-1} \right|^{1-nb/2} . \tag{6.17}$$

This reduces to previously obtained expressions for the special cases where these metrics agree with those considered elsewhere. In particular, it vanishes for extremal black-branes, for which $k = 1$ and $r_- = r_+$. For the four-dimensional Schwarzschild-type solution presented in section 2, we have $r_- = 0$ and so $T = |k|(n-1)/(4\pi r_+)$.

Entropy. The possibility of associating a temperature with a space-time involving horizons immediately suggests that it may also be possible to associate to it an entropy, using the thermodynamic relation

$$\frac{\partial S}{\partial(-M)} = \frac{1}{T} \,. \tag{6.18}$$

The unusual negative sign in this expression arises because of a technical complication in defining an entropy in the present instance. The complication arises because the entropy is associated with degrees of freedom behind the horizon, where the globally-defined time-like Killing vector changes direction. This situation is very much like what happens for the de Sitter space, for which the above expression is used to define the entropy [37].

For simplicity, we restrict first ourselves to the simplest Schwarzschild solution with $r_- = 0$, $k = -1$ and $n = 2$, for which we have $T = 1/(4\pi r_+)$ and $\mathcal{T}/V = M/V = -P/V_n = -r_+/2GV_n$, with V_n denoting the volume of the n-sphere. In this case, the entropy becomes:

$$\frac{\partial S}{\partial(-M)} = -8\pi G V_n \frac{M}{V}\,, \tag{6.19}$$

from which we integrate to find

$$\frac{S}{V} = 4\pi G V_n \left(\frac{M}{V}\right)^2, \tag{6.20}$$

where the integration constant is chosen to ensure $S(M=0) = 0$.

Note that, although both S and M both diverge due to infinite volume of the planar or the hyperbolic directions, the entropy and tension per unit volume are finite, and are related in the same way as are these quantities for a black-brane. Notice also that we retrieve the usual expression, $S = 4\pi G M^2$ when we specialize to the $k = 1$ case of a black brane.

In the general case, the expression for the entropy will depend on the electric charge as well. In order to extract the general form of the entropy we follow the standard prescription in terms of the euclidean action. Consider first the definition of the Gibbs free energy:

$$W = -T \log Z = TS + Q\Phi(r) - \mathcal{T}(r)\,, \tag{6.21}$$

where $\mathcal{T}(r)$ is given in eq. (4.5), while $\Phi(r)$ is the potential associated with the $q+2$ form. Note that we have here a sign change in the right-hand side of eq. (6.21) with respect to the usual definition of the free energy as explained above.

In the semiclassical approximation, we can identify the partition function Z with e^{-I_E}, where I_E corresponds to the euclidean action for the system From this fact we obtain immediately

$$TS + Q\Phi(r) - \mathcal{T}(r) = T I_E\,. \tag{6.22}$$

At this point, we need an expression for the euclidean action for our system. This takes the form

$$I_E = -\int d^d x \sqrt{g}\left(\alpha R - \lambda(\partial\phi)^2 - \eta e^{-\sigma\phi} F^2\right) - 2\alpha \int d^{d-1} x \sqrt{h} K\,. \tag{6.23}$$

where we have included the Gibbons-Hawking boundary action.

The contribution from a boundary at a surface of a fixed r is

$$\begin{aligned}\int d^{d-1}x\sqrt{h}K &= \frac{V(1-n)}{2T}\left[r_-^{n-1} - k\, r_+^{n-1} + (2 + A - (n-1)b) r_-^{n-1}\left(\left(\frac{r_+}{r}\right)^{n-1} - 1\right)\right], \\ &= \frac{\mathcal{T}}{4\alpha T}\,,\end{aligned} \tag{6.24}$$

where in the second equality we have used eq. (4.5). Consider now the solutions that we have found for our system. Following [37] we take the magnetic rather than the electric solution, using the duality transformations given in section 3.1. Substituting the solutions in eq. (6.23), it is straightforward to obtain a general expression for the euclidean action in terms of the parameters of the model. We then find the following expression that relates eq. (6.23) to other global quantities, and that allows interesting manipulations of eq. (6.22):

$$I_E = \frac{1}{2T}\left(\mathcal{T}(r) - Q\Phi(r)\right). \tag{6.25}$$

Substituting eq. (6.25) into eq. (6.22), a simple calculation yields the general relation

$$S = -I_E\,. \tag{6.26}$$

At this point, we can write the general expression for the entropy density s, using the known value of the temperature T, for any curvature k. Indeed, for $k = 1$, in which the entropy is calculated outside the outer horizon, it is enough to change sign on the last expression in eq. (6.21). We obtain the following compact form

$$s = \frac{S}{V} = \alpha 4\pi\, r_+^n \left| k - \left(\frac{r_-}{r_+}\right)^{n-1} \right|^{nb/2} = \frac{1}{4G}\sqrt{g_{nn}}\,\big|_{r_+}\,, \tag{6.27}$$

where in the last equation we have used $\alpha = (16\pi G)^{-1}$. It is remarkable that, for any k, the expression for the entropy does not depend on the coordinate r : g_{nn} corresponds to the determinant of the induced metric on the n spatial dimensions, and it is calculated at the horizon r_+. In case $k = 1$, we obtain the well-known relation

$$S = \frac{\mathcal{A}}{4G}\,, \tag{6.28}$$

where $\mathcal{A}$ is the area of the black-hole horizon. Again for $k = -1$ and 0, the area of the horizon is infinite but we can still consider the entropy per unit volume. It is worth noting that these quantities can be made finite by modding out the planar or the hyperbolic subspace by discrete subgroups of ISO(n) and SO($n-1,1$), respectively, as the operation would leave the volume of the horizon finite.

Particle detectors. Where is the thermal distribution of particles which are described by the temperature and entropy just defined? We propose an answer for this which follows a similar discussion for de Sitter space-time [38]. We follow here the argument as described in ref. [39].

The key lies in the observation that the two-point propagator in the static regions is periodic in imaginary time, as is seen by the above derivation of the euclidean periodicity. This implies that the transition rate, $R(i \to j)$, for the excitation of a simple particle detector from level i to level j satisfies

$$R(i \to j) = R(j \to i)e^{(E_j - E_i)/T}\,. \tag{6.29}$$

This is precisely the relation which these rates must have if they are to satisfy detailed balance in the presence of a thermal distribution of particles. One thus infers that when it is in a steady state, the detector responds as if it is in the presence of such a thermal distribution.

7. Discussion

We now summarize our results, and indicate some directions for future work.

7.1 Summary

In [4], a variety of solutions to the $d = (n+q+2)$-dimensional dilaton, metric, antisymmetric-tensor equations were found under the assumption that q-dimensional subspace is flat and n-dimensional subspace is maximally-symmetric with scalar curvature $kn(n-1)$, $k = 0, \pm 1$. Among the solutions, the $k = 1$ class were identified to be the black q-branes, first discussed in [4], but the $k = 0, -1$ classes were new. One of the objectives in the present work was to provide viable interpretations for the solutions belonging to the new classes.

We interpreted that these solutions describe a non-trivial field configuration produced by a pair of charged q-branes, whose world surfaces are the time-like singularities of the metric. The two q-branes carry equal but opposite-sign charges associated with the anti-symmetric tensor field. Tension of the two q-branes are equal, but are generically negative-valued for both $k = -1$ and $k = 0$.

With such an interpretation, we have found that the metric near the singularities is static (regions II and IV of figure 1), and describes the fields in the immediate vicinity of the q-branes. By contrast, the metric at late times becomes time-dependent, and we interpreted this as describing the expansion of the metric as the two q-branes interact each other. The metric must expand in this way because stability precludes negative-tension branes from being free to move relative to the other within a fixed background geometry. For $k = -1$, the metric at asymptotically late times becomes flat. Region III of the maximally-extended space-time describes the time-reversed process of the situation in region I.

We also have shown that observers moving along time-like geodesics can enter the static regions near the q-branes from the past time-dependent region III, and can pass out from there into the future time-dependent region I. These time-like geodesics are repelled by the time-like singularities, and hence do not hit them while evolving throughout the static regions.

The horizons of the space-time describe the set of events where observers make the transition into or out of the static regions near the q-branes. Before crossing the horizon out of the static regions, there is only one q-brane inside an observer's past light-cone. After crossing the horizon, however, both q-branes are in the observer's past light-cone (but not in the future light-cone).

The metric near the horizon resembles that of the S-brane as described in [5]. For the special case of 0-branes ($q = 0$), our solutions agree precisely with the S-branes. However the physical interpretation we are providing resembles more the one of [8] than that in [5]. The point is that in the original S-brane solution the attention was concentrated only on the time-dependent part of the geometry for which an spacelike "object", the horizon, was assigned physical properties such as charge. When considering the full spacetime geometry it is then clear that the physical object corresponds to the singularity which being in the static region allows for an unambiguous definition of charge and tension. The singularities then are similar to orientifold planes [8] in the sense of having negative tension and being constrained to a fixed point in the r, t coordinates ($r = 0$). In this sense they are free from potential instabilities in the same way as the orientifold planes are. However orientifold planes are well-defined objects in terms of a set of boundary conditions, which our solutions are not a priori constrained to obey. Furthermore an orientifold plane usually preserves part of supersymmetry. A preliminary study indicates that our solutions are non supersymmetric. Therefore we believe that these negative-tension branes represent a generalization of orientifold planes.

Certain aspects of our solutions also share features with those discussed recently in [8, 9], where time-like orbifolds of flat space-time were considered and potentially interesting cosmological evolution were obtained by compactifying some extra dimensions. On the other hand, our solutions do not have the problems recently raised for those orbifold geometries (see for instance the last reference in [9]), since they are obtained by solving Einstein's equations without the need of orbifold twists. In this regard, our solutions can be considered as an explicit yet relatively simple class of string theory backgrounds for addressing various issues raised therein such as stability, causal structure and particle/string productions.

A feature of the solutions of [4] is that they ought to be ubiquitous to a wide class of supergravity, low-energy string theories, and compactfications thereof. In case considered as solutions of string theories, they have well-defined domains of validity which follow from the requirements of weak coupling ($e^{\phi} \ll 1$) and small curvature ($\alpha' \partial^2 \ll 1$). In that case, various stringy corrections are expected to alter the regions close to the time-like singularities. On the other hand, regions near the horizons would be free from such corrections.

7.2 Future directions

There are two important issues which we were able to resolve only partially here. One is the question of the stability of these space-times (for a recent discussion see [40]). We have shown that the past horizons of these space-times are likely to be unstable in precisely the same manner as the inner horizon of the Reissner-Nordström black-hole. A more complete investigation of stability is obviously of considerable interest. The second is the question of quantum instability, and whether the time-dependent fields in regions I and III give rise to particle production. We have argued that there is a natural definition for the Hawking temperature for the static space-times near the q-branes, and this strongly suggests that this is associated with thermal radiation as seen by the static observers. A more detailed calculation of particle production is certainly desirable.

The time-dependent regions, I and III, of the space-times are also of considerable interest, because they may open up a new avenue for the cosmology of the early Universe. In the future time-dependent region I, the space-time exhibits expansion of the hyperbolic directions, and a past horizon without a past space-like singularity. Since region III corresponds to the time-reverse of region I, taken together the two regions I and III offer an interesting realization of a singularity-free cosmology, which bounces from a contracting to an expanding Universe. One is eventually interested in a realistic bouncing cosmology free from instabilities at the past horizon. A possible prescription ensuring the stability would be a periodic identification of the Killing coordinate, r, in the time-dependent regions.

An obvious obstacle to constructing a realistic cosmology out of the solutions eqs. (2.3) and (2.4) is that co-moving observers do not see a homogeneous and isotropic space. This objection needs not be fatal, as it may describe space-time during the very early Universe — perhaps during inflation – before the Universe is really required to be isotropic and homogeneous. Indeed, an attractive brane cosmology for these early epoch has been proposed by utilizing brane-antibrane interactions [1]. Alternatively, it may be the higher-dimensional solutions which are of cosmological relevance. After all, these space-times do have three-dimensional hyper-surfaces which are homogeneous and isotropic. In the specific metrics presented in [4], this usually requires that the radial coordinate, r, describes a compact direction. This would be problematic if the spacetime also includes the static regions because, there, the r-coordinate would correspond to a compact time direction. Having closed time-like curves, it may also lead to orbifold instabilities [9]. Certainly, a direction for future work is to explore whether a reasonable cosmology can be constructed free from these pathologies.

We are currently investigating the above issues, and will report results elsewhere.

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